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Dept. : CSE

Course : Linear Algebra

Course code : MAT 215

Ans to the Q: NO: 01

Ans: In linear algebra, the rank of a matrix A is dimension of the column Space (or the row Space) of A, & the nullity of a matrix A is the dimension of the kernel (null space) of A. The rank-nullity theorem states that for any $m \times n$ matrix A:

$$\text{rank}(A) + \text{Nullity } A = n,$$

Where n is the number of columns of A.

Let's analyze each based on the dimensions of A.

(a) A is 4×4

For a 4×4 matrix A:

- Largest possible rank: The maximum rank of A is 4, since rank can not exceed the smaller of the number of rows or columns.

P.T.O

largest possible rank = 4

- Smallest possible nullity: If the rank is 4, the nullity is $4-4=0$

smallest possible nullity = 0

(b) A is 3×5

For a 3×5 matrix A:

- Largest possible rank: The maximum rank of A is 3, As the rank can not exceed the number of rows (3 in this case)

largest possible rank = 3

- Smallest possible nullity: If the rank is 3, then by the rank-nullity theorem, the nullity is
$5-3=2$

Smallest possible nullity = 2

(c) A is 5×3

For a 5×3 matrix A:

• Largest possible rank: The maximum rank of A is 3, Since rank cannot exceed the number of columns (3 in this case)

largest possible rank = 3

• Smallest possible ~~rank~~ nullity: If the rank is 3, then by the rank-nullity theorem, the nullity is $3-3=0$

Smallest possible nullity = 0

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Anstothe! Q:NO: 02

Ans! Given that A is a 3×3 matrix & its null space is a line throught the Origin in 3-space, let's analyze the implications for the row & column spaces of A.

Understanding the null space :-

The null space of a matrix A consists of all vectors x such that $Ax = 0$.

- Since the null space of A is a lime thorughtg the Origin, It has dimension 1. This means there is exactly One free Variable & the rest are dependent on it.

According to the rank-nullity theorem:

$$\text{rank}(A) + \text{nullity}(A) = n$$

For our 3 × 3 matrix A, where $n = 3$:

$$\text{rank}(A) + \text{Nullity}(A) = 3$$

* Given that the nullity of A is 1 (since its null Space is a line through the Origin), we have:

$$\text{rank}(A) + 1 = 3 \Rightarrow \text{Rank}(A) = 2$$

* Row Space & column Space of A

• Rank & row space: The row space of A is the span of Its rows & has dimension equal to the rank of A. Since the rank of A is 2, the dimension of the row space is 2, which is a plane in 3- Space, not a line.

• Rank & column Space: The Column space of A is the span of its columns & also has dimension equal to the rank of A. Since the rank of A is 2, the dimension of the column Space is 2, which is a plane in 3 space, not a line.

The row Space or column Space of A cannot be a line through the origin. They are both planes in 3- dimensional Space.

Ans: to the Q: NO: 03

Ans: It looks like three may be a type in the characteristic polynomial you've provided. It currently reads as $p(\lambda) = \lambda^3 - 2\lambda^3 + \lambda + 5$. However, the polynomial has two λ^3 terms, which is unusual.

Assuming the polynomial you intended to write is $P(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$, we can proceed to find the determinant of matrix A.

Characteristic polynomial & Determinant -

For $a^3 + 3$ matrix A, the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$ is of the from $\lambda^3 - tr(A)\lambda^2 + c_1\lambda + \det(A)$.

Given the characteristic polynomial:

$$P(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5,$$

We can Identify the determinant det (A) as the constant term of the polynomial.

Therefore, the determinant of matrix A is

$$\det(A) = 5.$$

Ans) Let's analyze the dilation operator T defined on the space of 2×2 matrices, M_{22} with a dilation factor $k = 3$

Definition of the Dilation Operator:-

The dilation operator T with factor K scales each entry of a 2×2 matrix by K. Specifically, if A is a 2×2 matrix then,

$$T(A) = KA = 3A$$

Space & Dimensions:-

- The space M_{22} consist of all 2×2 matrices.
- The dimension of M_{22} is $2\times2 = 4$, so, the vector space M_{22} is 4-dimensional.

1. Rank of T

- The rank of T is the dimension of the image of T.
- Since T is a dilation operator, every 2×2 matrix is scaled version of the original.

• The rank of T is the dimension of the image of T is thus the entire space M_{22} ~~the~~ because any 2×2 matrix can be reached by scaling another matrix (specifically for any matrix B, you can find a matrix A such that $T(A) = B$ by choosing $A = \frac{1}{3}B$)

Therefore, the Image of T Spans the entire space M_{22} which has dimension 4.

So, the rank of T is:-

$$\text{Rank}(T) = 4$$

2. Nullity of T :-

• The nullity of T is the dimension of the kernel (null space) of T. The kernel of T consists of all matrices A such that $T(A) = 0$.

For $T(A) = 3A = 0$, it must be that $A = 0$ because the dilation factor 3 is non-zero. Thus, the only matrix that maps to the zero matrix under T is the zero matrix itself.

Therefore, the kernel of T contains only the zero matrix & its dimension (nullity) is 0.

So, the nullity of T is:-

$$\text{Nullity}(T) = 0.$$