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Ans 1) a) Rank is the dimension of column space since A is a 4×4 the maximum rank is 4. Nullity is the dimension of the null space if the rank is 4 then the nullity must be $4-4=0$
rank is 4 nullity is 0

Ans 1) b) Rank is the dimension of column space since A is a 3×5 matrix, the maximum rank is the smaller of the number of rows or columns which is 3 if the rank is 3, then the nullity must be $5-3=2$.
rank is 3 nullity $= 2$,

Ans 1) c) Rank is the dimension of the column space since A is 5×3 matrix the maximum rank is the smaller of the number of row which is 3
If the rank is 3, then the nullity must be $3-3=0$,
rank is 3 nullity is 0

Ans 2) No the row space or column space of a 3×3 matrix. A cannot be a line through the origin if the null space is a line through the origin The null space of A consists of all vectors x such that $Ax = 0$ if the null space is a line through the origin in 3 dimensional space it means the null space has dimension Therefore the rank of A is $3-1=2$ due to rank nullity theorem if the dimension of the null space equals the number of column since A is a 3×3 matrix which conclusion make a plane through origin not a line. The row space of A has the same dimension as the column space, which is again a plane through the origin not a line.

Ans 3) $P(\lambda) = \lambda^3 - 2\lambda^3 + \lambda + 5$

$P(\lambda) = -\lambda^3 + \lambda + 5$ (simplifying)

$P(\lambda) = -\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0$ (3×3 matrix A polynomial characteristics)

Ans 3) $P(\lambda) = -\lambda^3 + \lambda + 5$ (Simplified polynomial term $c_0 = 5$)

Therfore the determinant of matrix A is $det(A) = 5$.

Ans 4) Here the operator T is a dilation operator on the space M_{22} the space of all 2×2 matrices the dilation operator with factor $K = 3$ acts on any matrix $A \in M_{22}$ by multiplying it by 3

$$T(A) = 3A$$

Rank of T since $T(A) = 3$ and multiplying by 3 bijective operation the transformation doesnt change the dimensionality of the space therefore the image of T is the entire space $M22$. The space M_{22} has dimension 4 because a 2×2 matrix has 4 indipendent entries so the rank of $T = 4$.

The space of matrix A such that $T(A) = 0$ for $T(A) = 3A = 0$ the only solution is $A = 0$ therefore the null space of T contains only the zero matrix

so Nullity $(T) = 0$.