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Course : MAT-215

Ans to the Q.No-1

\* The Rank-Nullity Theorem:  
The rank nullity theorem states that for any matrix  $A$ , the rank of  $A$  (the dimension of the column space) equals the number of  $A$  or columns of  $A$ .

In summary

Matrix size	max Rank	min nullity
$4 \times 4$	4	0
$3 \times 3$	3	2
$5 \times 3$	3	0

Ans to the Q.No-2

② It is given in the question that the null space of  $A$  is a line thus it can be interpreted that it is 1-dimensional subspace consider the rank nullity theorem:

$$P(A) + \text{nullity}(A) = \dim(A)$$

Hence,

$P(A)$  is the rank of  $A$ .

putting the values in the above equation.

$$P(A) + 1 = 3$$

$$P(A) = 2$$

Hence it can be said that it is the dimension of its row or column space. Thus it can be said that column space cannot be a line, but it will be a plane.

Ans to the Q. No-3

<sup>2</sup>/<sub>7</sub> Relating the Determinant to the characteristic polynomial. the key to this problem lies in understanding the relationship between the determinant of a matrix and its characteristic polynomial.

We know that  $P(\lambda) = \det(A - \lambda I)$ . if we evaluate  $P(0)$  we get:  $\{$

$$P(0) = \det(A - 0I) = \det(A)$$

\* Evaluating the characteristic polynomial, we are given  $P(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$ . To find the determinant of  $A$ , we simply need to evaluate  $P(0)$ :

$$P(0) = (0)^3 - 2(0)^2 + (0) + 5 = 5.$$

Therefore, the determinant of matrix  $A$ , denoted as  $\det(A)$  is  $\dots 5$ .