Mid Assessment│Spring - 2024

Md. Shafayet Hossain

CSE - 21^{st} Batch │ **Course Code:** MAT - 329

Course Title: Mathematical Analysis for Computer Science

ID: 2121210071

Answer to the Question no- 1

(a)

Proposition:

A proposition is basically a statement that can be judged as either true or false. It's an idea presented for consideration, but in a way that allows it to be definitively right or wrong.

Here's an example: "Cats are mammals." This sentence expresses a proposition that is clearly true (assuming we agree on the definitions of "cat" and "mammal").

Propositions are different from other kinds of statements. For instance, an exclamation like "Wow, that pizza looks good!" isn't a proposition because it doesn't really say anything that can be objectively true or false. An order like "Please pass the salt" also isn't a proposition - it's an instruction, not a statement about the world.

(b)

Advantages of Digital Logic Design:

Digital Logic Design offers several advantages over traditional analog approaches:

- Accuracy and Reliability.
- Ease of Use.
- Noise Immunity.
- Storage and Transmission.
- Scalability and Integration.
- Programmability.

Answer to the Question no- 2

(a)

Predicate:

A predicate is a function that takes one or more inputs (often variables) and returns a Boolean value (true or false). It essentially asks a question about the input(s) and answers with a yes/no response.

Predicates are powerful tools because they allow us to express conditions and make decisions within algorithms and programs.

Here's an example of a predicate in mathematical analysis for computer science:

Predicate: isEven(x)
This predicate takes an integer x as input. It evaluates to true if x is even (divisible by 2) and false otherwise.

We can use this predicate in various ways:

- **Check if a number is even:**

```
def isEven(x):
  return x % 2 == 0

number = 10
if isEven(number):
  print(number, "is even")
else:
  print(number, "is odd")
```

- **Filter a list of numbers:**

```
numbers = [1, 4, 7, 9, 12]
even_numbers = list(filter(isEven, numbers))
print("Even numbers:", even_numbers)
```

In this example, the filter function iterates through the numbers list and keeps only the elements that satisfy the isEven predicate, resulting in a list of even numbers.

(b)

Here's a proof for the statement "If r is irrational, then $\sqrt{r}$ is also irrational" using proof by contrapositive:

1. State the contrapositive:

The contrapositive of the original statement is: "If $\sqrt{r}$ is rational, then r is rational." This means we will prove that if the square root of r is rational (can be expressed as a fraction), then r itself must also be rational.

2. Assume the opposite of the conclusion:

Assume for the sake of contradiction that $\sqrt{r}$ is rational. This means $\sqrt{r}$ can be expressed as a fraction in its simplest form, p/q, where p and q are integers with no common factors ($q \neq 0$).

3. Derive a contradiction:

Squaring both sides of the equation $\sqrt{r} = p/q$, we get:

$r = (\sqrt{r})^2 = (p/q)^2$

This simplifies to: $r = p^2/q^2$

Here, p^2 and q^2 are both integers (squares of integers), making r a rational number expressed as the fraction p^2/q^2.

4. Reach a contradiction:

This contradicts our initial assumption that r is irrational.

5. Conclusion:

Since assuming $\sqrt{r}$ to be rational leads to a contradiction, the original statement must be true. Therefore, if r is irrational, then $\sqrt{r}$ is also irrational.

Answer to the Question no- 3

Here's the proof that the standard deviation of a sequence of values $X_1 \dots X_n$ is zero if all the values are equal to the mean:

1. **Definition of Standard Deviation:**

   Standard deviation (SD) is a measure of how spread out the data is from its average value (mean). It's calculated by finding the squared deviations from the mean, averaging them, and then taking the square root.

2. **Deviation from the Mean:**

   If all the values (X_1) are equal, let's call this common value "M" (the mean). In this case, the deviation of each value from the mean would be:

   $X_1 - M = M - M = 0$

   This holds true for all values in the sequence ($X_2, X_3, \dots, X_n$) because they are all equal to M.

3. **Squared Deviations:**

   Since the deviations from the mean are all zero, the squared deviations from the mean will also be zero for each value $(X_1 - M)^2 = 0^2$.

4. **Average Squared Deviations:**

   When calculating the standard deviation, we average the squared deviations from the mean. If all the squared deviations are zero, their average will also be zero.

5. **Standard Deviation:**

   Finally, the standard deviation is calculated by taking the square root of the average squared deviations. Since the average squared deviations are zero, the square root of zero will be zero.

   Therefore, the standard deviation of a sequence of values $X_1 \dots X_n$ is zero if all the values are equal to the mean.

   In essence, if all the data points are clustered at a single point (the mean), there's no spread in the data, and hence the standard deviation is zero.