Victoria University of Bangladesh

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Ans to the qu: No: 1

(1) $y = e^{3x+2}$

$y_1 = \frac{dy}{dx} = e^{3x+2} \frac{d}{dx}(3x+2)$

$= e^{3x+2}(3(1)+0)$

$= 3e^{3x+2}$

$y_2 = \frac{d^2y}{dx^2}$

$= 3\left[\frac{d}{dx}\left(e^{3x+2}\right)\right]$

$= 3\left(3e^{3x+2}\right]$

$= 9e^{3x+2}$

$= 9y$ Ans:

(2)

$y = \log x + a^x$

$y_1 = \frac{dy}{dx} = \frac{1}{x} + a^x \log a \left[\because \frac{d}{dx}(a^x) = a^x \log a\right]$

$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) + \log a \frac{d}{dx}(a^x)$

$= \frac{-1}{x^2} + (\log a)(a^x \log a)$

$= \frac{-1}{x^2} + (a^x \log a)^2$

Ans!

Ans to the Qu; No; 2

(a) $f(x) = 3x^2 - 2x + 4$ at $x = 0$ and 3.

Find the derivative of $f(x)$:

$3x^2 - 2x + 4$

$f'(x) = 6x - 2$

find the slopes of the tangent lines at $x = 0$ and $x = 3$

At $x = 0$: $f'(0) = 6(0) - 2 = -2$

At $x = 3$: $f'(3) = 6(3) - 2 = 16$

Now, we have the slopes of the tangent lines ~~at~~ at $x = 0$ and $x = 3$.

use the point-slope form of a linear equation to write the equations of the tangent lines:

At $x = 0$

point: $(0, f(0)) = (0, 4)$

slope: $m = -2$

Equation of the tangent line: $y-y_1=m(x-x_1)$

plug in the values: $y-4=-2(x-0)$

simplify: $y-4=-2x$

Final equation of the tangent line at $x=0$: y

$y=-2x+4$

At $x=3$;

point: $(3, f(3))=(3, 31)$

slope: $m=16$

Equation of the tangent line: $y-y_1=m(x-x_1)$

plug in the values: $y-31=16(x-3)$

simplify: $y-31=16x-48$

final equation of the tangent line at $x=3$: $y=16x-17$

so, the equation's of the tangent lines to the function $f(x)=3x^2-2x+4$ at $x=0$ and $x=3$ are:

At $x=0$: $y=-2x+4$

At $x=3$: $y=16x-17$ — Ans:

Ans to the QU:NO: 3

To find the second derivative of y with respect to x, given the differential equation:

$$\frac{d^2y}{dx^2}+3\frac{d^2y}{dx}=4y=xe^x$$

Let's first differentiate both the sides of the equation with respect to x to isolate the second derivative term:

$d/dx[d^2y/dx^2]+d/dx[3*(d^2/dx)]-d/dx[4y]$
$=d/dx[xe^x]$

Now, let's break down each term:

1. $d/dx[d^2y/dx^2]=d^3y/dx^3$
2. $d/dx[3(d^2/dx)]=3(d^3y/dx^3)$
3. $d/dx[4y]=4(dy/dx)$
4. $d/dx[xe^x]=e^x+xe^x$

Now, let's substitute these derivatives back into the

$d^3y/dx^3+3(d^3y/dx^3)-4(dy/dx)=e^x+xe^x$

combine like terms:

$4(d^3y/dx^3)-4(dy/dx)=e^x+xe^x$ Ans:

Ans to the qu:No: 4

$y = x^2 - 2x + 3$

$\frac{dy}{dx} = 2x - 2x = 0$

$x = \frac{2}{2}$

$x = 1$

so the point is given as

$y = x^2 - 2x + 3$

$= (1)^2 - 2 \cdot 1 + 3$

$= 1 - 2 + 3$

$= 4 - 2$

$= 2$

so the point is $(1, 2)$ Ans;

Ans to the qu. No: 5

Given $w = \cos(x^2+2y) - e^{4x - z^4 y} + y^3$

partial derivative with respect to x

$\left(\frac{dw}{\partial x}\right)$:

$\frac{\partial w}{\partial x}\left(\cos(x^2+2y) - e^{4x-z^4y} + y^3\right)$

Applying the chain rule for the first term:

$-\sin(x^2+2y)\frac{d}{dx}(x^2+2y) - 4e^{4x} - 0 + 0$

simplifying:

$-2y\sin(x^2+2y) - 4e^{4x}$

partial derivative with respect to y

$\left(\frac{dw}{\partial y}\right)$:

$\frac{\partial}{\partial y}\left(\cos(x^2+2y) - e^{4x-z^4y} + y^3\right)$

Applying the chain rule for the first term:

$-2\sin(x^2+2y)-0-z^4+3y^2$

simplifying : $-2\sin(x^2+2y)-z^4+3y^2$,

partial derivative with respect to

$z\left(\frac{\partial w}{\partial z}\right)$

$\frac{\partial}{\partial z}\left(\cos(x^2+2y)-e^{4x}-z^4y+y^3\right)$

since z appears only in the third term, its derivative with respect to z is: $-4zy^3$

so, the partial derivatives of w are:

$\frac{\partial w}{\partial x}=-2y\sin(x^2+2y)-4e^{4x}$

$\frac{\partial w}{\partial y}=-2\sin(x^2+2y)-z^4+3y^2$

$\frac{\partial w}{\partial z}=-4zy^3$ Ans.