Victoria University
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Final Assessment

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Answer to the question no 1

1. $y = e^{3x+2}$

$\therefore y_1 = \frac{dy}{dx} = e^{3x+2} \frac{d}{dx}(3x+2)$

$= e^{3x+2}(3(1)+0)$

$= 3e^{3x+2}$

$y_2 = \frac{d^2y}{dx^2} = 3\left[\frac{d}{dx}(e^{3x+2})\right]$

$= 3[3e^{3x+2}]$

$= 9e^{3x+2}$

$= 9y$ Ans.

2) $y = \log x + a^x$

$y_1 = \frac{dy}{dx} = \frac{1}{x} + a^x \log a \quad [\because \frac{d}{dx}(a^x) = a^x \log a]$

$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) + \log a \frac{d}{dx}(a^x)$

$= \frac{-1}{x^2} + (\log a)(a^x \log a)$

$= \frac{-1}{x^2} + a^x(\log a)^2$ Ans.

Answer to the question no 2

Ans:- $f(x) = 3x^2 - 2x + 4$ at $x = 0$ and 3

Step 1: Find the derivative of $f(x)$:-

$f(x) = 3x^2 - 2x + 4$

$f'(x) = \frac{d}{dx}(3x^2 - 2x + 4)$

using the power rules

$f'(x) = 6x - 2$

Step 2:- Find the slopes at $x=0$ and $x=3$

a) at $x = 0$

$f'(0) = 6(0) - 2$
$= -2$

ⓑ at $x = 3$

$f'(3) = 6 \times 3 - 2$
$= 18 - 2$
$= 16$

Now we have the slopes for the tangents:-

at $x = 0$, slope $= -2$
at $x = 3$, slope $= 16$

Step 3:- use the point-slope from to find the equations of the tangents:-

a) for $x = 0$

using the point-slope from where (x_1, y_1) is the point of tengenc,

$y - y_1 = m(x - x_1)$

plugging the values:-

$y - f(0) = (-2)(x - 0)$ (since $x_1 = 0$ and $y_1 = f(0)$)

$y - 3(0)^2 - 2(0) + 4 = -2x$

$y - 4 = -2x$

Now simplify:-

$y = -2x + 4$

so the equation of the tengent at $x=0$ is $y=-2+4$.

b) for $x=3$

using point slope again

$$y-y_1 = m(x-x_1)$$

plugging the values:-

$y-f(3) = (16)(x-3)$ [since $x_1=3$ and $y_1 = f(3)$]

$$y-(3(3)^2-2(3)+4) = 16(x-3)$$

$$y-(27-6+4) = 16(x-3)$$

$$y-25 = 16x-48$$

now simplify

$$y = 16x-23$$

so the equation of the tengent at $x=3$ is $y=16x-23$.

Answer to the question no 3

Given equation is $\frac{d^2y}{dx^2}+3\frac{d^2y}{dx}-4y=xe^x$

~~list deta~~

Take the list derivative of both side with equality

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)+\frac{d}{dx}\left(3\frac{d^2y}{dx}\right)-\frac{d}{dx}(4y)=\frac{d}{dx}(x.e^x)$$

$$\Rightarrow \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)+3\frac{d}{dx}\left(\frac{d^2y}{dx}\right)-4\frac{dy}{dx}=e^x+xe^x$$

$$\Rightarrow \frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)+3\frac{d^2}{dx^2}\left(\frac{d^2y}{dx}\right)-4\frac{d^2y}{dx^2}=e^x+xe^x$$

$$=\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}-4\frac{d^2y}{dx^2}=e^x+xe^x$$

So the second order differential equation for y with respect to x is.

$$\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}-4\frac{d^2y}{dx^2}=e^x+xe^x \text{. Ans.}$$

Answer to the question no 4

Ans:- $y = x^2 - 2x + 3$

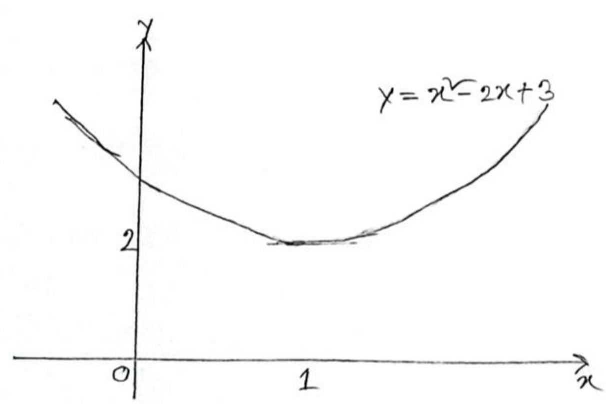
if $y = x^2 - 2x + 3$ then $\frac{dy}{dx} = 2x - 2$ and $\frac{d^2y}{dx^2} = 2$

Now $\frac{dy}{dx} = 2x - 2 = 0$ when $x = 1$

Since $\frac{d^2y}{dx^2} = 2 > 0$ for all values of x, this stationary point is a local minium. thus the function

$y = x^2 - 2x + 3$ has a local minium at the point $(1, 2)$

The below figure ~~on~~ shows the function $y = x^2 - 2x + 3$, with the local minium point at $(1,2)$ clearly visible.



Answer to the question no 5

$$w = \cos(x^2+2y) - e^{4x-z^4y} + y3$$

Ans:- $\frac{dw}{dx} = -\sin(x^2+2y).2x - e^{4x-z^4y}.4 + 0$

simplyfying

$$\frac{dw}{dx} = \cancel{2\sin} - 2x\sin(x^2+2y) - 4e^{4x-z^4y}$$

partial derivative with respect to y

$$\frac{dw}{dy} = -\sin(x^2+2y).2 + e^{4x-z^4y}.z^4 + 0$$

simplying

$$\frac{dw}{dy} = -2\sin(x^2+2y) + z^4e^{4x-z^4y}$$

partial derivative with respect to z:.

$$\frac{dw}{dz} = 0 - e^{4x-z^4y}.(-4yz^3) + 0$$

simplying

$$\frac{dw}{dz} = 4yz^3e^{4x-z^4y}$$

so the partial derivatives of w with respect to x, y and z are

$$\frac{dw}{dx} = -2x\sin(x^2+2y) - 4e^{4x-z^4y}$$

$$\frac{dw}{dy} = -2\sin(x^2+2y) + z^4e^{4x-z^4y}$$

$$\frac{dw}{dz} = 4yz^3e^{4x-z^4y}$$

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