

Victoria University of Bangladesh

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Course Title: Differential Equation and Fourier Analysis-scaled

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Ans to the Que No-1

① given,

$$y = e^{3x+2}$$

$$y_1 = \frac{dy}{dx} = e^{3x+2} \frac{dy}{dx} (3x+2)$$

$$= e^{3x+2} (3 \cdot 1 + 0)$$

$$= 3e^{3x+2}$$

$$y_2 = \frac{y_1 y}{dx^2} = 3 \left[\frac{d}{dx} (e^{3x+2}) \right]$$

$$= 3 (3e^{3x+2})$$

$$= 3 (3e^{3x+2})$$

$$= 9e^{3x+2}$$

$$= 9y \text{ (Ans.)}$$

P.T.O

(ii)

$$y = \log x + ax$$

$$\frac{dy}{dx} = \frac{1}{x} + a$$

$$\therefore y_1 = \frac{1}{x} + a$$

$$\frac{y_2 y}{dx^2} = \frac{d}{dx} \left(\frac{1}{x} + a \right)$$

$$= \frac{-1}{x^2}$$

$$y_2 = -\frac{1}{x^2} \quad \underline{\underline{\text{Ans.}}}$$

Ans. to the Que No-2

$$f(x) = 3x^2 - 2x + 4$$

$$f'(x) = \frac{d}{dx}(3x^2) - \frac{d}{dx}(2x) + \frac{d}{dx} \cdot 4$$

$$= 3x^2 - 2$$

at, $x=0$ Sep.

$$m = f'(0) = 3 \cdot 0^2 - 2$$

$$= -2$$

$$f(0) = 3 \cdot 0^2 - 2 \cdot 0 + 4$$

So, the point is (0, 4)

So the equation is —

$$y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 4 = -2(x - 0)$$

$$\Rightarrow y = -2x - 4$$

$$\left[\begin{array}{l} m = -2 \\ x = 0 \\ y = 4 \end{array} \right]$$

Ans to the que NO-4

$$y = x^2 - 2x + 3$$

$$\Rightarrow \frac{dy}{dx} = 2x - 2$$

Now, $2x - 2 = 0$
 $x = 1$

put $x = 1$ at y

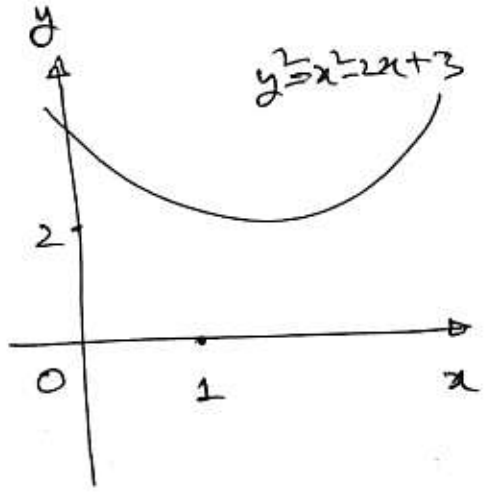
$$y = 1^2 - 2 \cdot 1 + 3$$

$$= 1 - 2 + 3$$

$$= 2$$

$$\therefore (x, y) = (1, 2)$$

Ans.



Ans to the Que No - 5

$$w = \cos(x^2 + 2y) - e^{4x - 2^4y} + y^3$$

$$\frac{dw}{dx} = \frac{d}{dx} \left\{ \cos(x^2 + 2y) - e^{4x - 2^4y} + y^3 \right\}$$

$$= \sin(x^2 + 2y) \cdot (2x) - (e^{4x - 2^4y}) \cdot 4$$

$$= -2x \sin(x^2 + 2y) - 4e^{4x - 2^4y}$$

$$\frac{dw}{dy} = \frac{d}{dy} \left\{ \cos(x^2 + 2y) - e^{4x - 2^4y} + y^3 \right\}$$

$$= -\sin(x^2 + 2y) \cdot 2 + 3y^2$$

$$= -2 \sin(x^2 + 2y) + 3y^2$$

$$\frac{dw}{dy} = 4 \cdot 2^3y e^{(4x - 2^4y)}$$

Ans to the que No-3.

to find the second order differential equation for 'y' with respect to 'x', will first differentiate the given equation with respect to 'x' multiple time.

Given equation,

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - 4y = x \cdot e^x$$

lets differentiate it step by step.

① Take the first derivative of both sides with respect to

x :-

$$\frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) + \frac{d}{dx} \left(3 \frac{dy}{dx} \right) - \frac{d}{dx} (4y) = \frac{d}{dx} (x \cdot e^x)$$

$$\Rightarrow \frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) + 3 \frac{d}{dx} \left(\frac{dy}{dx} \right) - 4 \frac{dy}{dx} = e^x + x e^x$$

$$\Rightarrow \frac{d^2}{dx^2} \left(\frac{d^2y}{dx^2} \right) + 3 \frac{d^2}{dx^2} \left(\frac{dy}{dx} \right) - 4 \frac{dy}{dx} = e^x + x e^x$$

$$\Rightarrow \frac{d^4y}{dx^4} + 3 \frac{d^3y}{dx^3} - 4 \frac{dy}{dx} = e^x + x e^x$$

So the

Q.E.D

⑥

So, the second-order differential equation for y with respect to x is :

$$\frac{d^4 y}{dx^4} + 3 \frac{d^3 y}{dx^3} - 4 \frac{d^2 y}{dx^2} = e^x + x e^x.$$

Ans.

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