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Course Title : Differential Equation and Fourier Analysis

Course Code : MAT-325

"Final Exam"

Ans to the Q No. 1

Given function,

$f = x^2 - 2x + 3$ or, $y = x^2 - 2x + 3 \ldots$ (1)

Now,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 2x + 3)$$

$$= \frac{d}{dx}(x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(3)$$

$$= 2x - 2 + 0$$

$$= 2x - 2 \ldots \text{(2)}$$

Now,

$2x - 2 = 0$

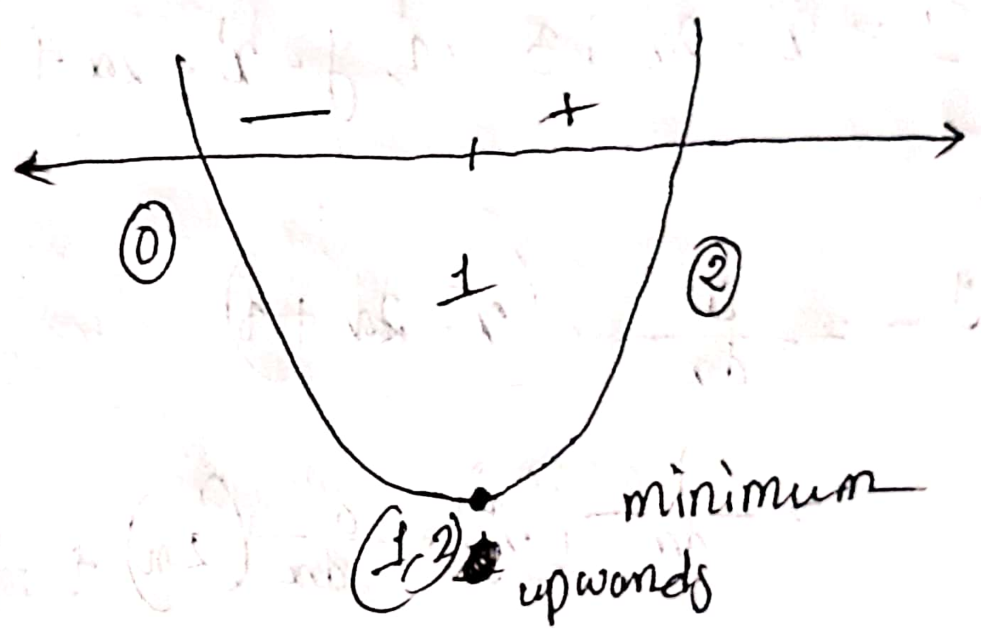
$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

$\therefore x = 1$

2

nature of the points:



~~Now we are putting $x=2$ we find,~~ in equation (2) ~~we find,~~

$$2x-2$$
$$=2\times(2)-2$$
$$=4-2$$
$$=2$$

then, we ~~can~~

Now, we are putting $x = 2$ in equation (2) then we can find,

$$2x - 2$$
$$= 2 \times (2) - 2$$
$$= 4 - 2$$
$$= 2$$

Again, we are putting $x = 0$ in equation (2) then can find,

$$2x - 2$$
$$= 2 \times (0) - 2$$
$$= 0 - 2$$
$$= -2$$

After putting $x = 2$ and $x = 0$ in equation (2) we can find the nature of the point.

Then, we can find the y value,

From, equation (1) we can find,

$$y = x^2 - 2x + 3$$
$$= (1)^2 - 2\times 1 + 3 \quad [\because x = 1]$$
$$= 1 - 2 + 3$$
$$= 4 - 2 = 2$$

minimum point at (1, 2).

stationary Poin at (1, 2).

~~Ans-to-the-Q-No-8~~

~~Given,~~

$$\cancel{w = \cos(x^2+2y) - e^{4x-2^4 y} + y^3}$$

~~Now, By partial derivating we can find,~~

$$\cancel{\frac{d}{dx}(w) = \frac{d}{dx}}$$

Ans-to-the-Q-No-2

Given,

$$f(x) = 3x^2 - 2x + 4$$

Now,

$$y = f(x)$$

Now, given function is -

$$y = 3x^2 - 2x + 4$$

At,

$$x = 0,\ y = 3 \times (0)^2 - 2 \times (0) + 4$$

$y = 4$

Point $(0, 4)$ is common both to the curve and the tangent.

At $x = 0$ the slope of curve is the same as slope of the tangent.

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 2x + 4)$$

$= 6x - 2$ is the slope of the curve.

~~At point~~ $x = 0$ ~~$x = 0$ the slope of the curve is~~

At ~~$x = 0$~~ $x = 0$ the slope of the curve is

At $x = 0$, $m = (6 \times 0) - 2$

$= -2$

The tangent passes through the point $(0, 4)$; its slope is -2.

The equation of the tangent is -

$y = mx + c$

$y = (-2) \times 0 + c$

$\Rightarrow 4 = 0 + c$

$\Rightarrow c = 4$

$\therefore y = -2x + 4$ ------ (i)

Again, for $x=3$,

Given,

$$f(x) = 3x^2 - 2x + 4$$

Now function is,

$$y = 3x^2 - 2x + 4$$

At, $x=3$,

$$y = 3\times(3)^2 - 2\times(3) + 4$$

$$= 27 - 6 + 4$$

$$= 31 - 6$$

$$= 25$$

Point $(3, 25)$ is common both to the curve and the tangent.

At $x=3$ the slope of the curve is same as the slope of the tangent.

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 2x + 4)$$

$= 6x - 2$ is the slope of the curve.

At, $x=3$ the slope of the curve is,

At, $x=3$, $m = (6 \times 3) - 2$

$= 18 - 2$

$= 16$

The tangent passes through the point ~~(3, 25)~~ (3, 25).

its slope is 16.

The equation of the tangent is,

$y = mx + c$

$\Rightarrow 25 = (16 \times 3) + c$

$\Rightarrow 25 = 48 + c$

$\Rightarrow 25 - 48 = c$

$\Rightarrow c = -23$

$\therefore c = -23$

$\therefore y = 16x - 23$ ---- (ii)

For, $x = 0$ the tangent of the equation for the given function it is

$y = -2x + 4$ ——— (i)

Again for $x = 3$ the tangent of the equation for the given function is

$y = 16x - 23$ ——— (ii)

Ans.

Ans to the Q No 1

(i)

$y = e^{3x+2}$

$y_1 = \frac{dy}{dx} = \frac{d}{dx}\left(e^{3x+2}\right)$

$= e^{3x+2} \frac{d}{dx}(3x+2)$

$= e^{3x+2}(3.1+0)$

$= 3e^{3x+2}$

$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(3e^{3x+2}\right)$

$= 3\left[\frac{d}{dx}\left(e^{3x+2}\right)\right]$

$= 3\left(3e^{3x+2}\right)$

$= 9e^{3x+2}$

$= 9y$

~~(Ans)~~ (Ans.)

(ii)

Given

$y = \log x + ax$

$\frac{dy}{dx} = \frac{d}{dx}(\log x + ax)$

$= \frac{d}{dx}(\log x) + \frac{d}{dx}(ax)$

$\therefore y_1 = \frac{1}{x} + a$

Again

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x} + a\right)$

$= \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}(a)$

$$= \cancel{\frac{d}{dx}\left(x^{-1}\right) - 1}$$

$$= \frac{d}{dx}\left(x^{-1}\right) + 0$$

$$= -x^{-1-1}$$

$$= -x^{-2}$$

$$y_2 = -\frac{1}{x^2}$$

(Ans.)

Ans. to the Q. No. 5

Given,

$\cancel{W = \cos(x^2+2y) - e^{4x}}$

$W = \cos(x^2+2y) - e^{4x - z^4 y} + r^3$

$\frac{dW}{dx} = \frac{d}{dx}\{\cos(x^2+2y) - e^{4x-z^4y} + r^3\}$

$= -\sin(x^2+2y).(2x) - (e^{4x-z^4y}).4$

$= -2x\sin(x^2+2y) - 4e^{4x-z^4y}$

$\frac{dW}{dy} = \frac{d}{dy}\{\cos(x^2+2y) - e^{4x-z^4y} + \cancel{r^3}\ y^3\}$

$= -\sin(x^2+2y).2 + \cancel{3r^2}\ 3y^2$

$= -2\sin(x^2+2y) + \cancel{3r^2}\ 3y^2$

16

$\cancel{= 2\sin}$

$= 2\sin(x^2+2y) + 3y^2$

$= \frac{dw}{dz} = \frac{d}{dz}\left\{\cos(x^2+2y) - e^{4x-z^4y} + y^3\right\}$

$\cancel{= 4z^3y\, e}$

$= 4z^3y\, e^{4x-z^4y}$

Ans.

Ans- to the - Q-No-3

To find the second order differential equation for "y" with respect to "x" will first differentiate the given equation with respect to "x" to multiple time:

Given example,

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = xe^x$$

Let's differentiate it step by step.

~~#8~~ 18

(1) Take of the first derivative of both sides with respect to x:

$$\cancel{\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)+\frac{d}{dx}\left(3\frac{d^2y}{dx^2}\right)-}$$

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)+\frac{d}{dx}\left(3\frac{d^2y}{dx^2}\right)-\frac{d}{dx}(4y)=\frac{d}{dx}\left(x\cdot e^{x}\right)$$

$$\cancel{\Rightarrow \frac{d^2}{dx^2}}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)+3\frac{d}{dx}\left(\frac{d^2y}{dx}\right)-4\left(\frac{dy}{dx}\right)=e^{x}+xe^{x}$$

$$\Rightarrow \frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)+3\frac{d^2}{dx^2}\left(\frac{d^2y}{dx}\right)-4\frac{d^2y}{dx^2}=e^{x}+xe^{x}$$

$$\Rightarrow \frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}-4\frac{d^2y}{dx^2}=e^{x}+xe^{x}$$

So, the second-order differential equation for y with respect to x is :-

$$\frac{d^4 y}{dx^4} + 3\frac{d^3 y}{dx^3} - 4\frac{d^2 y}{dx^2} = e^x + xe^x$$

Ans.