victoria university of Bangladesh

Mid- Assessment

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Ans to the qu: No: 1

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

solution:

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

By using Rationalization,

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x \to 0} \frac{(1-x)-1}{x(\sqrt{1+x}+1)}$$

$$= \frac{1}{\sqrt{1}+1}$$

$$= \frac{1}{2}$$ Ans.

Ans to the qu; No; 2

Given function : $f(x) = x^3 + 5x^2$

Derivative of x^3 with respect to x :

$$\frac{d}{dx}(x^3) = 3x^{3-1}$$
$$= 3x^2$$

Derivative of $5x^2$ with respect to x : $\frac{d}{dx}(5x^2) = 2 \cdot 5x^{2-1} = 10x$

Now, combining the derivatives of the individual terms:

$$f'(x) = \frac{d}{dx}(x^3) + \frac{d}{dx}(5x^2) = 3x^2 + 10x$$

So, the derivative of $f(x) = x^3 + 5x^2$ with respect to x is $f'(x) = 3x^2 + 10x$.

Ans!

Ans to the question 3

The integral provided is :-

$\int (2e^{x} + \frac{6}{x} + \ln 2)\, dx$

Let's integrate each term separately:

1. $\int 2e^{x}\, dx$:

The integral of e^{x} with respect to x is simply e^{x}. so, the integral of $2e^{x}$ with respect to x is $2e^{x}$.

2. $\int \frac{6}{x}\, dx$:

The integral of $1/x$ with respect to x is $\ln|x|$. Therefore, the integral of $6/x$ with respect to x is $6 * \ln|x| + c$, where c is the constant of integration.

3. $\int \ln(2)\, dx$:

The integral of a constant such as $\ln(2)$ with respect to x is simply x times the constant, so the integral of $\ln(2)$ with respect to x is $\ln(2)x$.

putting it all together, the integral of the given expression is:

$2e^{x} + 6 * \ln|x| + \ln(2)x + c$

where c is the constant of integration.

Ans to the qu:No: 4

The function $f(x) = x^2 \sin x$

$(uv)' = u'v + uv'$

In your case, $u(x) = x^2$ and $v(x) = \sin x$

Let's find the derivaties of $u(x)$ and $v(x)$:

$u'(x) = 2x$ (derivative of x^2 with respect to x)

$v'(x) = \cos(x)$ (derivative of $\sin(x)$ with respect to x)

Now apply the product rule:

$f'(x) = (x^2 \sin(x))' = u'v + uv' = (2x)(\sin(x)) + (x^2)(\cos(x))$

so, the derivative of $f(x) = x^2 \sin(x)$ with respect to x is:

$f'(x) = 2x \sin(x) + x^2 \cos(x)$ **Ans:**

Ans to the Qu: No: 5

The chain Rule is a fundamental theorem in calculus that deals with finding the derivative of a composite function. It allows you to compute the rate of change of a composition of functions by relating the derivatives of the individual functions involved. In simple terms, it helps you find how the change in one variable affects the change in another variable through a chain of connected functions.

Mathematically, if you have a composition of two functions, $f(g(x))$, where $g(x)$ is the inner function and $f(x)$ is the outer function, the chain Rule states that the derivative of this composition can be expressed as the product of the derivative of the inner function with respect to x:

$$\frac{d}{dx}[f(g(x))] = f'(g)(x)) \cdot g'(x).$$

Here:

* $f'(g(x))$ is the derivative of the outer function f with respect to its variable, evaluated at $g(x)$.

* $g'(x)$ is the derivative of the inner function g with respect to x.

The chain Rule is crucial for calculating derivatives of complex functions that are built from simpler functions. It is ~~applications~~ applicable in various areas of mathematics, physics, engineering, economics, and many other fields where rates of change play a significant role.