Victoria University
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MID Term Assessment

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Answer to the question no 1

1. Find the $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$, it is like $\frac{0}{0}$

Rationalise numerator

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$\lim_{x \to 0} \frac{(\sqrt{1+x})^2-(1)^2}{x(\sqrt{1+x}+1)}$$

$$\lim_{x \to 0} \frac{1+x-1}{x(\sqrt{1+x}+1}$$

$$\lim_{x \to 0} \frac{x}{x(\sqrt{1+x}+1}$$

$$\lim_{x \to 0} \frac{1}{\sqrt{1+0}+1}$$

$$\lim_{x \to 0} \frac{1}{1+1} = \frac{1}{2}$$

$$\therefore \lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} = \frac{1}{2} \text{ Ans.}$$

Answer to the question no 2

2. Find the derivative of $f(x) = x^3 + 5x^2$

we have, using the above theorems

$$f'(x) = \frac{d}{dx}(x^3+5x^2)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(5x^2) \text{ (derivative of sum)}$$

$$= \frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) \text{ (derivative of a constant multiple)}$$

$$= 3x^2 + 10x \quad \left(\because \frac{d}{dx}(x^3) = 3x^2,\ \frac{d}{dx}(x^2) = 2x\right)$$

$\therefore f(x) = x^3 + 5x^2$ to $f'(x) = 3x^2 + 10x$

Ans.

Answer to the question no 3

3. Find the integral $\int(2e^x+\frac{6}{x}+\ln 2)\,dx$

$$\int(2e^x+\frac{6}{x}+\ln 2)\,dx$$

$$=2\int e^x dx+6\int\frac{1}{x}dx+\ln 2\int dx$$

$$=2e^x+6\ln|x|+(\ln 2)x+c$$

Answer

Answer to the question no 4

4. if $f(x)=x^2\sin x$. Find $f'(x)$.

Solution:-

$$f'(x)=\frac{d}{dx}(x^2\sin x)$$

$$=x^2\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x^2) \;\ldots\text{ Product Rule}$$

$$=x^2(\cos x)+\sin x\frac{d}{dx}(x^2) \;\ldots\text{ Formula for } \sin x$$

$$=x^2\cos x+\sin x(2x) \;\ldots\ldots\text{ power Rule}$$

$$\therefore f(x)=x^2\cos x+2x\sin x$$

Ans.

Answer to the question no 5

5. Describe chain theorem 1.

Answer:- The chain rule:-

The chain rule allows us to differentiate the composition of two functions. Recall from the module function 11 that the composition of two functions g and f is

$$(f\circ g)(x) = f(g(x)).$$

we start with x, apply g, then apply f. the chain rule tell us low to differentiate such as function.

Theorem (chain rule):-

Let f, g be differentiable functions. then the derivative of their composition is $\frac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$.

In Lebibniz notation, we may write $u = g(x)$ and $y = f(u) = f(g(x))$; diagrammatically $x \xrightarrow{g} u \xrightarrow{f} y$.

then the chain rule says that differentials cancel in the sense that

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}.$$

proof:- To calculate the derivative, we must evaluate the limit

$$\frac{d}{dx}[f(g(x))] = \lim_{\Delta x \to 0} \frac{f(g(x+\Delta x)) - f g(x)}{\Delta x}.$$

The trick is to multiply and divide by an extra term in the expression above, as shown, so that we obtain two expressions which both express ~~th~~ rate of change:

$$\frac{f(g(x+\Delta x)) - f(g(x))}{\Delta x} = \frac{f(g(x+\Delta x)) - f(g(x))}{g(x+\Delta x) - g(x)}\;\frac{g(x+\Delta x) - g(x)}{\Delta x}$$

we can then rewrite the desired limit as

$$\left(\lim_{\Delta x\to 0}\frac{f(g(x+\Delta x))-f(g(x))}{g(x+\Delta x)-g(x)}\right)\left(\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}\right).$$

The ratio in the first limit expresses the change in the function f, from the value at $g(x)$ to its value at $g(x+\Delta x)$, relative to the difference between $g(x+\Delta x)$ and $g(x)$. so as $\Delta x\to 0$, this first term approaches the ~~deb~~ derivative of f at the point $g(x)$, namely $f'(g(x))$. the second limt is clearly $g'(x)$. we conclude that

$$\frac{d}{dx}[fg(x)] = f'(g(x))\,g'(x),$$

as required.

The proof above is not entirely rigorous: for instance, if there are values of Δx close to zero such that $g(x+\Delta x)-g(x)=0$, then we have division by zero in the first limit. However, a fully rigorous proof is beyond the secondary school level.

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