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Course Title : Differential Equation and Fourier Analysis

Course Code : MAT-325

Ans to the Q No 1

Given,

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$$= \frac{\sqrt{1+0} - 1}{0}$$

$$= \frac{1-1}{0}$$

= অনির্ণেয়

Ans.

Ans. to the Q No-2

Given
$f(x) = x^3 + 5x^2$

Now,

$$\frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}\left(x^3 + 5x^2\right)$$

$$= \frac{d}{dx}\left(x^3\right) + \frac{d}{dx}\left(5x^2\right)$$

$$= x^2 + 5x$$

Ans.

Ans - to - the - Q - No - 3

Given,

$$\int \left(2e^{x} + \frac{6}{x} + \ln 2\right) dx$$

$$= \int 2e^{x}\, dx + \int \frac{6}{x}\, dx + \int \ln 2\, dx$$

$$= 2e^{x} + 6 \ln|x| + \frac{1}{2} + c$$

Ans.

Ans to the Q No 4

Given,

$f(x) = x^2 \sin x$

$\frac{d}{dx}(f(x)) = \frac{d}{dx}(x^2 \sin x)$

$\Rightarrow \frac{d}{dx}(f(x)) = x^2 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^2)$

$\Rightarrow f'(x) = x^2 \cos x + \cancel{\sin x\, 2x}\ \sin x \cdot 2x$

$\Rightarrow f'(x) = x^2 \cos x + 2x \sin x$

$\therefore f'(x) = x^2 \cos x + 2x \sin x$

Ans.

Ans to the Q No. 5

Chain Theorem :

Given $a \in R$ and functions f and g such that g is differentiable at a and f is differentiable at $g(a)$, then

$$(f \circ g)'(a) = f'(g(a))\, g'(a)$$

We start with a proof which is entirely correct, but contains in it the heart of the argument.

Proof By definition

$$(f \circ g)'(a) = \lim_{h \to 0} \frac{f(g(a+h)) - f(g(a))}{h}$$

We rewrite this in the following way:

$$(f \circ g)'(a) = \lim_{h \to 0} \frac{f(g(a)+d_h) - f(g(a))}{h} \quad (*)$$

where $d_h = g(a+h) - g(a)$. ~~Note~~ Note that $d_h \to 0$ as $h \to 0$. We are now in a pretty good situation. $(*)$ looks extremely similar to the definition of $f'(g(a))$. Since it is of the form

$$\frac{f(g(a)+\text{tiny}) - f(g(a))}{\text{tiny}}$$

The difficulty is that the two tiny quantities are tending to 0 at different rates. Fortunately we can get around this by ~~...~~ using the same trick we've been using in our earlier examples. we multiply by a clever form of 1. More precisely, we rewrite (1) in the form

$$(f \circ g)'(a) = \lim_{h\to 0} \frac{f(g(a)+d_h) - f(g(a))}{d_h} \cdot \frac{d_h}{h}$$

$$= \left(\lim_{h\to 0} \frac{f(g(a)+d_h) - f(g(a))}{d_h}\right)\left(\lim_{h\to 0} \frac{d_h}{h}\right)$$

$$= \left(\lim_{d_h\to 0} \frac{f(g(a)+d_h) - f(g(a))}{d_h}\right)\left(\lim_{h\to 0} \frac{g(a+h) - g(a)}{h}\right)$$

$$= f'(g(a))\,g'(a)$$

At first glance, the above argument seems sound. Upon closer inspection, however, there are two potential issues:

(1) We multiplied by $\frac{\delta_n}{\delta_n}$. This is only possible if $\delta_n \neq 0$ for all $h \neq 0$ in some neighbourhood of 0.

(2) We replaced $\lim_{h \to 0}$ by $\lim_{\delta_n \to 0}$.

We now write down a proof of the chain rule which resolves both of these issues.

We begin by rewriting the function of $(f \circ g)'(a)$ in terms of d_h:

$$(f \circ g)'(a) = \lim_{h \to 0} \frac{f(g(a)+d_h) - f(g(a))}{h} \quad (2)$$

Next,

$$f(g(a)+d_h) - f(g(a)) = \phi(h) \cdot d_h \quad (3)$$

where

$$\phi(h) = \begin{cases} \dfrac{f(g(a)+d_h) - f(g(a))}{d_h} & \text{if } d_h \neq 0 \\ f'(g(a)) & \text{if } d_h = 0 \end{cases} \quad (4)$$

plugging (3) into (2) we find

$$(f \circ g)'(c) = \lim_{h \to 0} \phi(h) \cdot \frac{d_h}{h}$$

$$= \left(\lim_{h \to 0} \phi(h)\right) \cdot \left(\lim_{h \to 0} \frac{d_h}{h}\right)$$

$$= \left(\lim_{h \to 0} \phi(h)\right) \cdot g'(c)$$

To conclude the proof of the chain Rule, it therefore remains ~~that~~ only to show that

$$\lim_{h \to 0} \phi(h) = f'(g(c)).$$