Victoria University of Bangladesh

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Answer to the question no 1(a)

 # Describe Universal set Empty set and Disjoint set?

Answer : Universal set: All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the universal set which we denote by

U

unless otherwise stated or implied.

Given a universal set U and a property P, there may not be any elements of U which have property P. For example, the following set has no elements:

S = {x | x is a positive integer, x2 = 3}

Empty set: Such a set with no elements is called the empty set or null set and is denoted by

∅

There is only one empty set. That is, if S and T are both empty, then S = T , since they have exactly the same elements, namely, none.

The empty set ∅ is also regarded as a subset of every other set. Thus we have the following simple result which we state formally.

Disjoint set :Two sets A and B are said to be disjoint if they have no elements in common. For example, suppose

A = {1, 2}, B = {4, 5, 6}, and C = {5, 6, 7, 8}

Then A and B are disjoint, and A and C are disjoint. But B and C are not disjoint since B and C have elements in common, e.g., 5 and 6. We note that if A and B are disjoint, then neither is a subset of the other (unless one is the empty set).

Answer to the question no 1(b)

 # Suppose S is a disjoint Union of finite sets A and B and S is a finite and n(S)=n(A)+n(B) prove that.

Answer:

Proof.

In counting the elements of $A \cup B$, first count those that are in A.

There are n(A) of these. The only other elements of $A \cup B$ are those that are in B but not in A. But since A and B are disjoint, no element of B is in A, so there are n(B) elements that are in B but not in A.

Therefore, $n(A \cup B) = n(A) + n(B)$.

For any sets A and B, the set A is the disjoint union of $A \setminus B$ and $A \cap B$.

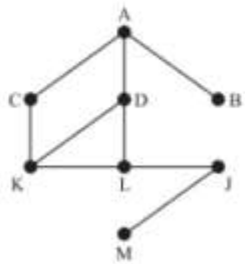
Thus Lemma 1.6 gives us the following useful result.

Answer to the question no 2(a)

 # Define Breadth-first Search Algorithm?

Answer:

This section discusses two important graph algorithms which systematically examine the vertices and edges of a graph G. One is called a depth-first search (DFS) and the other is called a breadth-first search (BFS). Other graph algorithms will be discussed in the next chapter in connection with directed graphs. Any particular graph algorithm may depend on the way G is maintained in memory. Here we assume G is maintained in memory by its adjacency structure. Our test graph G with its adjacency structure appears in Fig. 8-30 where we assume the vertices are ordered alphabetically.



(a)

Vertex	Adjacency list
A	B, C, D
B	A
C	A, K
D	A, K, L
J	L, M
K	C, D, L
L	D, J, K
M	J

(b)

During the execution of our algorithms, each vertex (node) N of G will be in one of three states, called the status of N, as follows:

STATUS = 1: (Ready state) The initial state of the vertex N.

STATUS = 2: (Waiting state) The vertex N is on a (waiting) list, waiting to be processed.

STATUS = 3: (Processed state) The vertex N has been processed.

The waiting list for the depth-first seach (DFS) will be a (modified) STACK (which we write horizontally with the top of STACK on the left), whereas the waiting list for the breadth-first search (BFS) will be a QUEUE.

(Breadth-first Search): This algorithm executes a breadth-first search on a graph G beginning with a starting vertex A.

Step 1. Initialize all vertices to the ready state (STATUS = 1)

Step 2. Put the starting vertex A in QUEUE and change the status of A to the waiting state (STATUS = 2).

Step 3. Repeat Steps 4 and 5 until QUEUE is empty.

Step 4. Remove the front vertex N of QUEUE. Process N, and set STATUS (N) = 3, the processed state.

Step 5. *Examine each neighbor J of N.*

(a) If STATUS (J) =1 (ready state), add J to the rear of QUEUE and reset STATUS (J) = 2 (waiting state).

(b) If STATUS (J) = 2 (waiting state) or STATUS (J) = 3 (processed state), ignore the vertex J.

[End of Step 3 loop.]

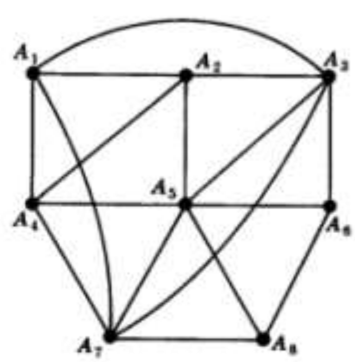
Step 6. *Exit.*

Answer to the question no 2(b)

 # Describe Welch-powell Algorithm?

Answer:

(a) Consider the graph G in Fig. 1 . We use the Welch-Powell Algorithm 8.4 to obtain a coloring of G. Ordering the vertices according to decreasing degrees yields the following sequence: A5, A3, A7, A1, A2, A4, A6, A8



The first color is assigned to vertices A5 and A1. The second color is assigned to vertices A3, A4, and A8. The third color is assigned to vertices A7, A2, and A6. All the vertices have been assigned a color, and so G is 3-colorable. Observe that G is not 2-colorable since vertices A1, A2, and A3, which are connected to each other, must be assigned different colors. Accordingly, $\chi(G) = 3$.

(b) Consider the complete graph Kn with n vertices. Since every vertex is adjacent to every other vertex, Kn requires n colors in any coloring. Thus $\chi(Kn) = n$.

There is no simple way to actually determine whether an arbitrary graph is n-colorable. However, the following theorem (proved in Problem 8.19) gives a simple characterization of 2-colorable graphs.

Algorithm 8.4 (Welch-Powell): The input is a graph G.

Step 1. Order the vertices of G according to decreasing degrees.

Step 2. Assign the first color C1 to the first vertex and then, in sequential order, assign C1 to each vertex which is not adjacent to a previous vertex which was assigned C1.

Step 3. Repeat Step 2 with a second color C2 and the subsequence of noncolored vertices.

Step 4. Repeat Step 3 with a third color C3, then a fourth color C4, and so on until all vertices are colored.

Step 5. Exit.

Answer to the question no 3(a)

 # : Suppose a Student is selected at random from 100 student s where 40 are taking mathematics, 30 are taking chemistry and 20are taking mathematics and chemistry. Find the probability that the students is taking mathematics or chemistry?

Answer

Let

M = {students taking mathematics} and

C = {students taking chemistry}. Since the space is equiprobable,

$P(M) = 40/100 = 4/10$, $P(C) = 30/100 = 3/10$, $P(M \text{ and } C) = P(M \cap C) = 20/100 = 1/5$

Thus, by the Addition Principle (Theorem 7.4), $p = P(M \text{ or } C) = P(M \cup C) = P(M) + P(C) - P(M \cap C) = 4/10 + 3/10 - 1/5 = 1/2$

Ans: 1/2 students is taking mathematics or chemistry

Answer to the question no 3(b)

 # Find an ordinary deck of 52 playing card?

Answer :

Let a card be selected from an ordinary deck of 52 playing cards. Let

A = {the card is a spade} and B = {the card is a face card}.

We compute P (A), P (B), and P (A ∩ B). Since we have an equiprobable space,

P (A) = number of spades number of cards = 13/52 = 1/4 ,

P (B) = number of face cards number of cards = 12/52 = 3/13

P (A ∩ B) = number of spade face cards number of cards = 3/52

Theorems on Finite Probability Spaces

The following theorem follows directly from the fact that the probability of an event is the sum of the probabilities of its points.

Theorem 7.1: The probability function P defined on the class of all events in a finite probability space has the following properties:

[P1] For every event A, $0 \le P(A) \le 1$.

[P2] P (S) = 1.

[P3] If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

The next theorem formalizes our intuition that if p is the probability that an event E occurs, then 1 − p is the probability that E does not occur. (That is, if we hit a target p = 1/3 of the times, then we miss the target 1 − p = 2/3 of the times.)

Theorem 7.2: Let A be any event. Then P (Ac) = 1 − P (A).

The following theorem (proved in Problem 7.13) follows directly from Theorem 7.1.

Theorem 7.3: Consider the empty set and any events A and B. Then:

(i) P (∅) = 0.

(ii) $P(A \setminus B) = P(A) - P(A \cap B)$.

(iii) If $A \subseteq B$, then $P(A) \le P(B)$.

Observe that Property [P3] in Theorem 7.1 gives the probability of the union of events in the case that the events are disjoint. The general formula (proved in Problem 7.14) is called the Addition Principle. Specifically:

Theorem 7.4 (Addition Principle): For any events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Answer to the question no 4(a)

 # Find the number m of seven-letter words that can be formed using the Letters of the word "BENZENE"?

Answer :

We seek the number of permutations of 7 objects of which 3 are alike (the three E's), and 2 are alike (the two N's). By Theorem 5.6,

$$m = P(7; 3, 2) = 7!/3!2! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1/3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 420$$

Ordered Samples

Many problems are concerned with choosing an element from a set S, say, with n elements. When we choose one element after another, say, r times, we call the choice an ordered sample of size r. We consider two cases.

(1) Sampling with replacement

Here the element is replaced in the set S before the next element is chosen. Thus, each time there are n ways to choose an element (repetitions are allowed). The Product rule tells us that the number of such samples is:

$$n \cdot n \cdot n \cdots n \cdot n (r \text{ factors}) = n^r$$

(2) Sampling without replacement

Here the element is not replaced in the set S before the next element is chosen. Thus, there is no repetition in the ordered sample. Such a sample is simply an r-permutation. Thus the number of such samples is:

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = n!/(n-r)!$$

Answer to the question no 4(b)

 # A farmer buys cows,3 pigs and 4 hens from a man who has 6 cows ,5 pigs,and 8 hens.Find the number m of choices that the farmer has?

Answer :

The farmer can choose the cows in C(6, 1) ways, the pigs in C(5, 3) ways, and the hens in C(8, 4) ways. Thus the number m of choices follows:

$m = (6/1)(5/3)(8/4) = (6/1).(5 \cdot 4 \cdot 3)/(3 \cdot 2 \cdot 1).(8 \cdot 7 \cdot 6 \cdot 5)/(4 \cdot 3 \cdot 2 \cdot 1) = 6 \cdot 10 \cdot 70$
$= 4200$

Answer: m = 4200