Victoria University
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Final Assessment

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Answer to the question No 1. (i)

i) $y = e^{3x+2}$

$y_1 = \frac{dy}{dx} = e^{3x+2} \frac{d}{dx}(3x+2)$

$= e^{3x+2}(3(1)+0)$

$= 3e^{3x+2}$

$y_2 = \frac{d^2y}{dx^2} = 3\left[\frac{d}{dx}(e^{3x+2})\right]$

$= 3[3e^{3x+2}]$

$= 9e^{3x+2}$

$= 9y$

ii) $y = \log x + ax$

$y_1 = \frac{dy}{dx} = \frac{1}{x} + a^x \log a \quad \left[\because \frac{d}{dx}(a^x) = a^x \log a\right]$

$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) + \log a \frac{d}{dx}(a^x)$

$= \frac{-1}{x^2} + (\log a)(a^x \log a)$

$= \frac{-1}{x^2} + a^x(\log a)^2$

Answer to the question No 2

a) The gradient of the tangent to a curve at any particular point is given by the derivative of the curve at that point.

so if $y = 3x^2 - 2x + 4$

$$\frac{dy}{dx} = 6x - 2$$

Now substitute $x=0$ into $f'(x)$:

$$f'(0) = 6(0) - 2$$
$$= -2$$

so the slope of tangent at $x=0$ is -2

using the point-slope from of a linear equation, where (x_1, y_1) is the given point and m is the slope

$$y - y_1 = m(x - x_1)$$

plugging in the values for the point $(0, f(0)) = (0, 4)$:

$$y - 4 = (-2)(x - 0)$$
$$y - 4 = -2x$$

Therefore the equation of the tangent at $x=0$ is $y = -2x + 4$

Tangent at $x = 3$

similarly, we need to find the slope of the tangent at $x=3$

taking the derivative of $f(x)$ with repect to x:

$$f'(x) = \frac{d}{dx}(3x^2 - 2x + 4)$$
$$= 6x - 2$$

Now substitute $x=3$ into $f'(x)$:

$$f'(3) = 6(3) - 2$$
$$= 16$$

so the slope of the tangent at $x=3$ is 16

using the point slope from of a linear equation, where (x_1, y_1) is the given point and m is the slope

$$y - y_1 = m(x - x_1)$$

plugging the values for the point $(3, f(3))$:

$$y - (3^2 - 2(3) + 4) = 16(x-3)$$
$$y - 11 = 16(x-3)$$

simplying we get

$$y = 16x - 48 + 11$$
$$y = 16x - 37$$

Therefore the equation of the tangent at $x=3$ is $y = 16x - 37$.

Answer to the question NO 3

Given $\frac{d^2y}{dx^2} + 3\frac{d^2y}{dx} - 4y = xe^x$

step1: Differentiate both sides of the equation with respect to x

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2} + 3\frac{d^2y}{dx} - 4y\right) = \frac{d}{dx} xe^x$$

step 2:- apply the derivative rules to each term on the left side

$$\frac{d^2y}{dx^3} + 3\frac{d^3y}{dx^2} - 4\frac{dy}{dx} = e^x + xe^x$$

step 3:- Differentiate both sides again with respect to x

$$\frac{d}{dx}\left(\frac{d^3y}{dx^3} + 3\frac{d^3y}{dx^2} - 4\frac{dy}{dx}\right) = \frac{d}{dx}\left(e^x + xe^x\right)$$

step 4: apply the derivative rules to each term on the left side

$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} = e^x + xe^x + e^x$$

step 5: simplifying the right hand side

$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} = 2e^x + xe^x$$

Second derivative of y with respect to x in the given differential.

$$\frac{d^2y}{dx^2} = \frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 2e^x - xe^x$$

$$4y = \frac{d^2y}{dx^2} + 3\frac{d^2y}{dx} - xe^x$$

Answer to the question No 4

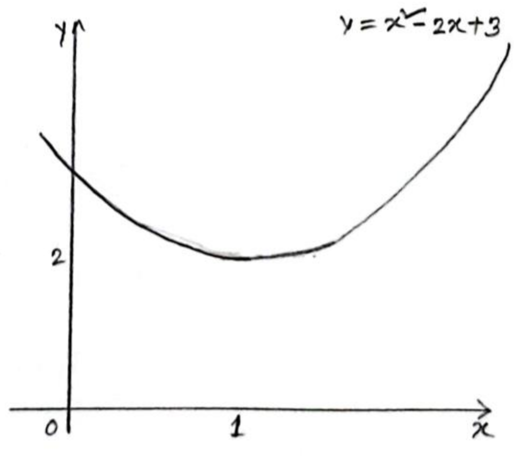
Solution:-

if $y = x^2 - 2x + 3$ then $\frac{dy}{dx} = 2x - 2$ and $\frac{d^2y}{dx^2} = 2$

Now $\frac{dy}{dx} = 2x - 2 = 0$ when $x = 1$

since $\frac{d^2y}{dx^2} = 2 > 0$ for all values of x, this satdionary point is a local minum; thus the function $y = x^2 - 2x + 3$ has a local minium at the point $(1, 2)$

the figure on the buttom shows the function $y = x^2 - 2x + 3$ with the local minium point at $(1, 2)$ clearly visible.



Answer to the question No 5

to find the partial derivatives of the function $w = \cos(x^2 + 2y) - e^{4x - z^4 y} + y^3$ we need

to differentiate with respect to each ~~of~~ variable seperately while treating the others as constants, lets calculate the partial delivatives:-

partial derivative with respect to x:.

$$\frac{\partial w}{\partial x} = -2x \sin(x^2 + 2y) - 4e^{4x - z^4 y}$$

partial derivative with respect to y

$$\frac{\partial w}{\partial y} = -2\sin(x^2 + 2y) + z^4 e^{4x - z^4 y} + 3y^2$$

partial derivative with respect to z:

$$\frac{\partial w}{\partial z} = 4yz^3 e^{4x - z^4 y}$$

These are the partial derivatives of the function w with respect to each variable $x, y,$ and z.

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