Victoria University of Bangladesh

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Answer to the question number : 01

(a)

All sets under investigation in any application of set theory are assumed to belong to same fixed large set called the universal set which we denote by 'U'.

Unless otherwise stated or implied.

Given a universal set U and a property P, there may not be any elements of U which have property P. For example, the following set has no elements:

$$S = \{x \mid x \text{ is a positive integer}, x^2 = 3\}$$

Such a set with no elements is called the empty set or null set and is denoted by $\emptyset$.

There is only one empty set. That is, if S and T are both empty, then $S = T$. Since they have exactly the same elements, namely none.

The empty set is also regarded as a subject of every other set. Thus we have the following simple result which we state formally.

Two sets A and B are said to be disjoint if they have no elements in common. For example:

$$A = \{1, 2\}, \quad B = \{4, 5, 6\} \quad \text{and} \quad C = \{5, 6, 7, 8\}$$

Then A and B are disjoint and A and C are disjoint. But B and C are not disjoint once B and C have elements in common, e.g 5 and 6. We not that, if A and B are disjoint, then neither is a subset of the other.

Answer to the question number : 1

(b)

Let

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{2, 3, 8, 9\}$

$A \cup C = \{1, 2, 3, 4\} \cup \{2, 3, 8, 9\}$

$= \{1, 2, 3, 4, 8, 9\}$

$- A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$- B \cup C = \{3, 4, 5, 6, 7\} \cup \{2, 3, 8, 9\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9\}$

Answer to the question number : 1

(a)

IdemPotent Laws	(1a) $A \cup A = A$	(1b) $A \cap A = A$
Associated Laws	(2a) $(A \cup B) \cup C = A \cup (B \cup C)$	(2b) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	(3a) $A \cup B = B \cup A$	(3b) $A \cap B = B \cap A$
Distributive Laws	(4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(4b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	(5a) $A \cup U = U$	(5b) $A \cap U = A$
Involution laws	(6a) $A \cup U = U$	(6b) $A \cap \emptyset = \emptyset$
Complements laws	(7a) $(A^c)^c = A$	(8b) $A \cap A^c = \emptyset$ (9b) $\emptyset^c = U$
Demorgan's laws	(10a) $(A \cup B)^c = A^c \cap B^c$	(10b) $(A \cap B)^c = A^c \cup B^c$

Answer to the question number - 02

(a)

we know that.

$$R = \{(1,2), (2,3), (3,3)\}$$
$$A = \{1,2,3\}$$

Then,

$$R^2 = R \circ R = \{(1,3), (2,3), (3,3)\} \text{ and}$$
$$R^3 = R^2 \circ R = \{(1,3), (2,3), (3,3\}$$

Accordingly,

$$\text{transitive } (R) = \{(1,2), (2,3), (3,3), (1,3)\}$$

Answer to the question number : 2

(b)

Equivalance Relation : Consider a nonempty set S. A Relation R and S is an equivalance relation if R is reflexive, symmetric and transitive. That is, R is an equivalance relation on S if it has the following three properties.

(i) For every $a \in S$, aRa.

(ii) If aRB, then bRa.

(iii) If aRb and bRc, then aRc.

The general idea behind an equivalance relation is that, it is a classification of objects which are in some way "alike". In fact the relation "=" of equality on any set S is a equivalance relation, that is:

(1) $a = a$ for every $a \in S$

(2) If $a = b$, the $b = a$

(3) If $a = b$, $b = c$, then $a = c$.

Answer to the question number : 03

(a)

Let s be the statement :

"It is not true that, roses are red and violets are blue". Then s can be written in the form $\neg(p \wedge q)$. However, as noted above, $\neg(p \wedge q) \equiv \neg p \vee \neg q$. Accordingly, s has the same meaning as the statement.

"Roses are not Red or violets are not blue."

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

FIG: 1.1

Answer to the question number : 3

(b)

Conditional and Biocontional statement :

Many statement particularly in mathematics are the form "if p then q. Such statements are called Conditional statements are denoted by $p \rightarrow q$.

The conditional $p - q$ is frequently read p implies q" or only q'.

Another common statement is of the form 'p if and only if q". Such statements are called bioconditional statements and are denoted by

$$p \leftrightarrow q$$

The truth values of $p \rightarrow q$ and $p \leftrightarrow q$ are defined by the tables in FIG. Observed that:

ⓐ The conditional $p \rightarrow q$ is false only when the first part p is true and the second part q is false. Accordingly, when p is false, the conditional $p \rightarrow q$ is true regardless of the truth value of q.

ⓑ The bioconditional $p \leftrightarrow q$ is true whenever p and q have the same truth values and false otherwise. The truth table of $\neg p \wedge q$ appears in FIG. (1.2). Note that, the truth table of $\neg p \vee q$ and $p \rightarrow q$ are identical, that is they are both false only in the second case. Accordingly, $p \rightarrow q$ is logically equivalent to $\neg p \vee q$; that is $p \rightarrow q \equiv \neg p \vee q$

In other words, the conditional statements "if p then q" is logically equivalant to the statement "Not p or q" which only involves the connectives $\vee$ and thats was already a part of our language. We may regard $p \to q$ as an abbreviation for an oft-recurring statement.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(a) $p \to q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(b) $p \leftrightarrow q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(c) $\neg p \vee q$

FIG - 1.2