Victoria University
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MID Term Assessment

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Answer to the question no 1

Answer to the question NO 1.

1) Answer

$$\int\left(\frac{1}{2x}-\frac{2}{x^2}+\frac{2}{\sqrt{x}}\right)dx$$

$$=2\int\frac{1}{x}dx - 2\int\frac{1}{x^2}dx+\frac{1}{2}\int\frac{1}{x}dx$$

Now solving.

$$\int\frac{1}{\sqrt{x}}dx$$

Apply power rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} \text{ with } n=-\frac{1}{2}$$

$$=2\sqrt{x}$$

Now solving

$$\int\frac{1}{x}dx$$

This is a standard integral

$$=\ln(x)$$

Now solving

$$\int\frac{1}{x^2}dx$$

Apply power rule with $n=-2$

$$=-\frac{1}{x}$$

plug in solved integrals

$$2\int\frac{1}{\sqrt{x}}dx+\frac{1}{2}\int\frac{1}{x}dx-2\int\frac{1}{x^2}dx$$

$$=\frac{\ln(x)}{2}+4\sqrt{x}+\frac{2}{x}$$

the problem is solved.

$$\int\left(\frac{2}{\sqrt{x}}+\frac{1}{2x}-\frac{2}{x^2}\right)dx$$

$$=\frac{\ln(x)}{2}+4\sqrt{x}+\frac{2}{x}+c \text{ (Answer)}$$

Answer to the question no 2

Answer to the question NO 2

2) $\int x^3 \ln x \, dx$

$\int n x^3 dx$

Apply linearity

$= n \int x^3 dx$

Now solving

$\int x^3 dx$

Apply power rule:

$\int x^n dx = \frac{x^{n+1}}{n+1}$ with $n=3$

$= \frac{x^4}{4}$

Plug in solved integrals

$n \int x^3 dx$

$= \frac{n x^4}{4}$

the problem is solved

$\int n x^3 dx$

$= \frac{n x^4}{4} + c$

Answer to the question no 3

Answer to the question 3

3) $\int 3y^2\sqrt{y^3-1}\,dy$

substitute $u = y^3-1 \rightarrow du = 3y^2dy$

$= \int \sqrt{u}\,du$

Apply power rule:

$\int u^n du = \frac{u^{n+1}}{n+1}$ with $n = \frac{1}{2}$

$= \frac{2u^{3/2}}{3}$

undo substitution $u = y^3-1$

$= \frac{2(y^3-1)^{3/2}}{3}$

the problem is solved

$\int 3y^2\sqrt{y^3-1}\,dy$

$= \frac{2(y^3-1)^{3/2}}{3} + C$ (Answer)

Answer to the question no 4

Answer to the question NO 4

4) Answer:-

$$\int_2^3 \frac{x^3-5x^2}{x}\,dx$$

$$\int \frac{x^3-5x^2}{x}\,dx$$

Expand.

$$=\int (x^2-5x)\,dx$$

apply linearity.

$$=\int x^2\,dx - 5\int x\,dx$$

Now solving.

$$\int x^2\,dx$$

apply power rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \text{ with } n=2$$

$$=\frac{x^3}{3}$$

Now solving.

$$\int x\,dx$$

apply power rule with $n=1$

$$=\frac{x^2}{2}$$

plug in solved integrals

$$\int x^2\,dx - 5\int x\,dx$$

$$=\frac{x^3}{3}-\frac{5x^2}{2}$$

problem is solved.

$$\int \frac{x^3-5x^2}{x}\,dx$$

$$=\frac{x^3}{3}-\frac{5x^2}{2}+c$$

$$=\frac{x^2(2x-15)}{6}+c \text{ (Answer)}$$

Answer to the question no 5

Answer:

Stokes' theorem is a fundamental theorem in vector calculus that relates the integral of a vector field over a surface to the line integral of the curl of the same vector field along the boundary of that surface. It is named after the mathematician George Gabriel Stokes who first formulated the theorem.

Mathematically, the statement of Stokes' theorem can be expressed as follows:

Let S be a smooth, oriented surface in three-dimensional space with boundary curve C. Let F be a continuously differentiable vector field defined on an open region containing S. Then, the line integral of the curl of F along C is equal to the surface integral of F over S:

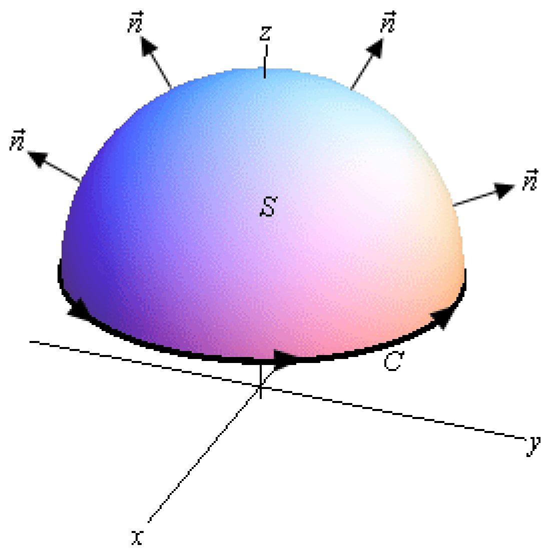
$\int_C F \cdot dr = \iint_S (\text{curl } F) \cdot dS$

Here, dr denotes the differential line element along the boundary curve C, dS denotes the differential surface element on the surface S, · denotes the dot product, and curl F is the curl of the vector field F.

Intuitively, the theorem states that the circulation of a vector field around a closed curve is equal to the amount of "rotation" or "curl" of the vector field over the surface bounded by that curve. Stokes' theorem is a powerful tool in physics and engineering for calculating quantities such as fluid flow, electromagnetic fields, and more.

In this section we are going to take a look at a theorem that is a higher dimensional version of Green’s Theorem. In Green’s Theorem we related a line integral to a double integral over some region. In this section we are going to relate a line integral to a surface integral. However, before we give the theorem we first need to define the curve that we’re going to use in the line integral.

Let’s start off with the following surface with the indicated orientation.



Around the edge of this surface we have a curve C. This curve is called the boundary curve. The orientation of the surface S will induce the positive orientation of C. To get the positive orientation of C think of yourself as walking along the curve. While you are walking along the curve if your head is pointing in the same direction as the unit normal vectors while the surface is on the left then you are walking in the positive direction on C.

Now that we have this curve definition out of the way we can give Stokes’ Theorem.

Stokes’ Theorem

Let S be an oriented smooth surface that is bounded by a simple, closed, smooth boundary curve C with positive orientation. Also let $\vec{F}$ be a vector field then,

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F}\cdot d\vec{S}$$

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