Name : Lagna Adhikary

ID : 2519170011

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Answer to the Question No - 1

$$* \int \left(\frac{1}{2x} - \frac{2}{x^2} + \frac{2}{\sqrt{x}}\right) dx$$

$$\int \frac{1}{2x}\, dx = \frac{1}{2} \ln(x) + c_1$$

$$\int \frac{-1}{x^2} = \frac{2}{x} + c_2$$

$$\int \frac{2}{\sqrt{x}}\, dx = 4\sqrt{x} + C_3$$

where, c_1, c_2, c_3 are constants of integration putting everything together,

$$\int \left(\frac{1}{2x} - \frac{2}{x^2} + \frac{2}{\sqrt{x}}\right) dx = \frac{1}{2} \ln(x) - \frac{2}{x} + 4\sqrt{x} + c.$$

(2) Answer to the Question No-2

* $\int x^2 \ln x \, dx$

To integrate $\int (x^2 \ln x)\, dx$,

let, $u = \ln x$

$dv = x^2 \, dx$

Then, $v = \frac{x^3}{3}$

$du = \frac{dx}{3}$

Hence,

$$\int u\, dv = uv - \int u\, du$$

$$\int x^2 . \ln x \, dx = \frac{x^3}{3} \cdot \ln x - \int \frac{x^3}{3} \cdot \frac{dx}{x}$$

$$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} \cdot dx$$

$$= \frac{x^3}{3} \cdot \ln x - \frac{3x}{9} + c$$

(3)

Answer to the Question No - 3

* $\int 3y^2\sqrt{y^3-1}\,dy$

$$\int 3y^2\sqrt{y^3-1}\,dy = \int \sqrt{y^3-1}\cdot 3y^2\,dy$$

$$= \int \sqrt{u}\,du$$

$$= \frac{2}{3}u^{3/2} + c$$

Hence,

$u = y^3 - 1$

then,

$\frac{du}{dy} = 3y^2$

$dy = \frac{du}{3y^2}$

Substituting back $u = y^3 - 1$,

$$\int 3y^2\sqrt{y^3-1}\,dy = \frac{2}{3}(y^3-1)^{\frac{3}{2}} + c$$

So, the substution is $u = y^3 - 1$

(4)

Answer to the Question No - 4

$$\int_2^3 \frac{x^3-5x^2}{x}\,dx$$

$$\int_2^3 \frac{x^3-5x^2}{x}\,dx = \int_2^3 (x^2-5x)\,dx$$

Now,

$$\int_2^3 (x^2-5x)\,dx = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2\right]_2^3$$

$$= \left\{\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2\right\} - \left\{\frac{1}{3}(2)^3 - \frac{5}{2}(2)^2\right\}$$

$$= \left\{\frac{1}{3}(27) - \frac{5}{2}(9)\right\} - \left\{\frac{1}{3}(8) - \frac{5}{2}(4)\right\}$$

$$= \left(9 - \frac{45}{2}\right) - \left(\frac{8}{3} - 10\right)$$

$$= \left(\frac{18-45}{2}\right) - \left(\frac{8-30}{3}\right)$$

$$= -\frac{27}{2} - \left(-\frac{22}{3}\right)$$

$$= -\frac{27}{2} + \frac{22}{3}$$

$$= \frac{-81+44}{6}$$

$$= \frac{-37}{6}$$

(5)

Answer to the Question No-5

* Stokes theorem : Stoke's theorem, also known as the kelvin Stokes theorem after Lord kelvin and George stokes. the fundamental theorem for curls or simply the curl theorem, is a theorem for curls or simply the curl on R^3. Given a vector field around the boundary of the surface. The classical theorem of stokes can be stated in one sentence: The line integral of a vector field over a loop is equal to the flux of its curl through the enclosed surface. It is illustrated in the figure, where the direction of positive circulation of the bounding contour $\partial\Sigma$, and the direction n of positive flux through the

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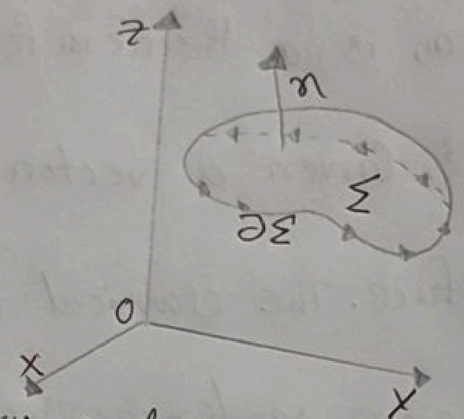
surface Σ, are related by a right-hand rule. For the right hand the fingers circulate along ∂Σ and the thumb is directed along n.



An illustration of stokes' theorem with surface Σ, its boundary ∂Σ and the normal vector n.

Stokes' theorem is a special case of the generalized stokes theorem. In particular, a vector field on R^3 can be considered as a 1-form in which case its curl is it's exterior derivative, a 2-form.