Final - Examination

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Answer to the question no :- 1.

Ans :-
The Newton - Raphson method is an iterative method for finding a root of an equation. Given an initial guess, x0, the method finds successively better approximations x1, x2, x3..... to the root by updating x_n to x_n+1 using the formula;

x_n+1 = x_n - f(x_n) / f'(x_n).

where f(x) is the equation whose root we want to find and f'(x) is its derivative.

For the equation x^4 - x - 10 = 0, we can use the Newton - Raphson method with an initial guess of x0 = 2:

$$x1 = x0 - \frac{(x0^4 - x0 - 10)}{(4x0^3 - 1)}$$

$$= 2 - \frac{(2^4 - 2 - 10)}{(4 \times 2^3 - 1)}$$

$$= 2 - \frac{(-4)}{7}$$

$$= 2.57142857.....$$

$$x2 = x1 - \frac{(x1^4 - x1 - 10)}{(4x1^3 - 1}$$

$$= 2.57142857... - \frac{(-0.857142857...)}{4.428571428....}$$

$$= 2.57142857.... + 0.195121951...$$

$$= 2.766550452.....$$

$$x3 = x2 - \frac{(x2^4 - x^2 - 10)}{(4x2^3 - 1)}$$

$$= 2.766550452..... - \frac{(-0.026582278...)}{4.932020725....}$$

Pg-2

$= 2.766550452 \ldots + 0.005339786 \ldots$

$= 2.771894838 \ldots,$

We can repeat the process until the desired accuracy is reached. The final result,

$x_3 = 2.771894838 \ldots$ is a root of the equation $x^4 - x - 10 = 0$.

Answer to the Question no - 2

The Gauss-Elimination method to solve the system of linear equations is as follows:-

Given the system;

$4x - y + 2z = 15$

$-x + 2y + 3z = 5$.

$5x - 7y + 9z = 8$

1. Start with the ~~fi~~ ~~1st~~ first equation and make the coefficient of x equal to 1, so divide the first equation by 4 to get;

$x - (\frac{1}{4})y + (\frac{1}{2})z = 3.75$.

2. Use this result to eliminate x in the second equation, add 4 times the first equation to the second equation:

$3y + 7z = -23$

3. Repeat this process to eliminate y in the third equation, add -5 times the second equation to the third equation;

$18z = 38$

4. Divide the third equation by 18 to get;

$z = 2$

5. Substitute z back into the second equation to get;

$3y + 7 \times 2 = -23$

$3y = -37$

6. Diviede the second equation by 3 to get;

$y = -13$

7. Substitute y and z back into the first equation to get;

$x - (\frac{1}{4}) \times (-13) + (\frac{1}{2}) \times (2) = 3.75.$

$x + (\frac{13}{4}) + 1 = 3.75.$

$x = -11.25.$

So the solution to the system of linear equations is $x =$ ~~11.25~~ -11.25, $y = -13$ and $z = 2$.

[Solved].

Answer to the Question NO-5

Ans:- Euler's method is a first-order numerical method used to solve ordinary differential equations (ODEs). It is based on the concept of approximating the solution at discrete intervals by using the value of the derivative of the solution at the current point.

The general form of the Euler's method is given by the following formula:

$$y(x+h) = y(x) + h * f(x, y(x))$$

where $y(x)$ is the approximation of the solution at x, h is the step size, and $f(x, y(x))$ is the right-hand side of the ODE.

For the given ODE, $\frac{dy}{dx} = x - y^2$, with $y(0)=1$, and $h=0.2$, the Euler's method can be used to find the approximation of y at $x=0.6$ as follows;

$$y(0.8) = y(0.6) + h \times (x - y12) \big|_{x=0.6,\, y=y(0.6)=1.}$$

$$= 1 + 0.2 \times (0.6 - 1)$$

$$= 0.6. \quad \text{[solved]}.$$

Answer to the Question no - 6

The Runge-Kutta method is numerical technique for solving ordinary ~~different~~ differential equations (ODEs). It uses iterative approximations to calculate the solution at a given time step, based on estimates at previous time steps.

The 4th order Runge-Kutta method involves four approximations, known as K1, K2, K3 and K4, to estimate the value of y at the next time step. The formula for the 4th order Runge- Kutta method is given as;

$$y(x+h) = y(x) + (h/6) \times (K1 + 2K2 + 2K3 + K4).$$

where h is the step size and K1, K2, K3 and K4 are given by;

$$K1 = h \times f(x, y(x)).$$

$$K2 = h \times f\left(x + \left(\frac{h}{2}\right), y(x) + \left(\frac{K1}{2}\right)\right)$$

$$K3 = h \times f\left(x + \left(\frac{h}{2}\right), y(x) + \left(\frac{K2}{2}\right)\right).$$

$$K4 = h \times f(x+h, y(x) + K3)$$

For the given differential equation,

$$\frac{dy}{dx} = y^2 - \frac{2x}{y^2 + x}$$

$y(0) = 1$,

At $x = 0.1$,

$$y1 = y(0) + \left(\frac{0.1}{6}\right) \times (K1 + 2K2 + 2K3 + K4).$$

At $x = 0.2$.

$$y2 = y1 + \left(\frac{0.1}{6}\right) \times (K1 + 2K2 + 2K3 + K4).$$

At $x = 0.3$,

$$y3 = y2 + \left(\frac{0.1}{6}\right) \times (K1 + 2K2 + 2K3 + K4).$$

To calculate K1, K2, K3 and K4, we need to evaluta the function $f(x, y(x))$ at various points. This requires multiple iterations of the Runge-Kutta method, with each iteration giving a more accurate estimate of y. The number of iterations will depend on the desired level of accuracy and the step size h.

In general, the Runge-Kutta method can be used to solve for y at any given x, but for this specific problem, the values of y at $x = 0.1, 0.2$, and 0.3 can be obtained using the process described above.

Answers to the Question no - 7

Solution :-

The value of table for x and y.

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.0588	0.0385	0.027

Using Weddle's Rule.

$$\int y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$$\int y\,dx = \frac{3\times 1}{10}\left[1 + 5\times 0.5 + 0.2 + 6\times 0.1 + 0.0588 + 5\times 0.0385 + 0.027\right].$$

$$\int y\,dx = 1.37349$$

Soloution by weddle's Rule is 1.37349.

[Solved].

"The End"