Victoria University of Bangladesh
Dept of Computer Science ~~of~~ &
Engineering, Semester:- Fall 2022
Program:- B.Sc in CSIT
Course title:- Numeric Methods
Course code:- MAT 415

Name:- Nusrat Jahan Tanshee
ID:- 2520200011
Batch:- 20

Final Assessment

1

Ans to the Q no-(1)

$\boxed{4}$ Let $f(x) = x^4 - x - 10$

Here,

$f(1) = -10 < 0$ and

$f(2) = 4 > 0$

$$\begin{array}{c} 2 \\ \uparrow \\ 1.8555 \\ \downarrow \\ 1 \end{array}$$

So, $f(x)=0$ has at least one root lies between 1 and 2.

Since,

$f(x) = x^4 - x - 10$

$\therefore f'(x) = 4x^3 - 1$

By Newton-Raphson method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$= \frac{4x_n^4 - x_n - x_n^4 + x_n + 10}{4x_n^3 - 1}$$

$$\therefore x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1} \quad \text{——— (i)}$$

2

Taking $x_0 = 1.9$ the relation (1) gives

For $n=0$, $x_1 = \dfrac{3x_0^4 + 10}{4x_0^3 - 1}$

$\Rightarrow x_1 = \dfrac{\cancel{3x(1.}}{}$

$\Rightarrow x_1 = \dfrac{3 \times (1.9)^4 + 10}{4 \times (1.9)^3 - 1}$

$\therefore x_1 = 1.8572$

For $n=1$, $x_2 = \dfrac{3x_1^4 + 10}{4}$

$\Rightarrow x_2 = \dfrac{3 \times (1.8572)^4 + 10}{4 \times (1.8572)^3 - 1}$

$\therefore x_2 = 1.85556$

For $n=2$, $x_3 = \dfrac{3x_2^4 + 10}{4x_2^3 - 1}$

$= \dfrac{3 \times (1.85556)^4 + 10}{4 \times (1.85556)^3 - 1}$

$\therefore x_3 = 1.85556$

(3)

Due to repetition of x_2 and x_3, we pause our work. Hence, the required root is 1.856 correct to three decimal places. (Ans.)

Ans to the Q no-(2)

Given the system of equation

$$\left.\begin{aligned} 4x-y-2z&=15 \\ -x+2y+3z&=5 \\ 5x-7y+9z&=8 \end{aligned}\right] \text{———} (i)$$

The system (i) is equivalent to $Ax=B$ — (ii)

Where,

$$A=\begin{bmatrix} 4 & -1 & -2 \\ -1 & 2 & 3 \\ 5 & -7 & 9 \end{bmatrix}, x=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} 15 \\ 5 \\ 8 \end{bmatrix}$$

Now our aim is to reduce the augmented matrix of (i) to upper triangular matrix.

Now the augmented matrix (A X B) is

$$\left[\begin{array}{ccc|c} 4 & -1 & -2 & 15 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right] \quad [r_1' = \tfrac{1}{4} r_1]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & -7/4 & 5/2 & 35/4 \\ 0 & -23/4 & 23/2 & -43/4 \end{array}\right] \quad \begin{array}{l} [\because r_2' = r_2 + r_1] \\ [\because r_3' = r_3 - 5r_1] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & -1 & 2 & -43/23 \end{array}\right] \quad \begin{array}{l} [\because r_2' = \tfrac{4}{7} r_2] \\ [\because r_3' = \tfrac{4}{23} r_3] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 24/7 & 504/161 \end{array}\right] \quad [\because r_3' = r_3 + r_2]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 1 & 21/23 \end{array}\right] \quad [\because r_3' = 7/24\, r_3]$$

The reduced system is

$$x - \frac{1}{4}y - 1/2z = 15/4$$

$$y + 10/7z = 5$$

$$z = 21/23$$

Now we back substitution, we get

$$z = 21/23$$

$$y = 5 - 10/\cancel{7} \times \cancel{21}^{3}/23$$

$$= 5 - \frac{30}{23} \quad \therefore y = \frac{85}{23}$$

$$x = \frac{1}{4} \times \frac{85}{23} + \frac{1}{2} \times \frac{21}{23}$$

$$= \frac{85}{92} + \frac{21}{46} = \frac{127}{92}$$

$$\therefore x = \frac{127}{92}; \; y = \frac{85}{23}; \; z = \frac{21}{23}$$ (Ans)

Ans to the Q no-3

(1) Here, $x_0=0$, $x_1=1$, $x_2=3$, $x_3=8$

$y_0=1$, $y_1=3$, $y_2=13$, $y_3=123$

$x-x_0=x$	$x_0-x_1=-1$	$x_0-x_2=-3$	$x_0-x_3=-8$
$x_1-x_0=1$	$x-x_1=x-1$	$x_1-x_2=-2$	$x_1-x_3=-7$
$x_2-x_0=3$	$x_2-x_1=2$	$x-x_2=x-3$	$x_2-x_3=-8$
$x_3-x_0=8$	$x_3-x_1=7$	$x_3-x_2=5$	$x-x_3=x-8$

By Lagranges interpolation formula, we have

$$y(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x-x_3)}y_1+$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$y(5)=\frac{(5-1)(5-3)(5-8)}{(-1)\times(-3)\times(-8)}\times 1+\frac{5\times(5-3)(5-8)}{1\times(-2)\times(-7)}\times 3+$$

$$\frac{5(5-1)(5-8)}{3\times2\times(-5)}\times 13+\frac{5\times(5-1)(5\times3)}{8\times7\times5}\times 123$$

$= 1-6.4286+26+17.57143$

$= 38.14286$

(Ans)

Ans to the Q. no- 7

Represent the distance x along x-axis and y along y-axis then $y_0, y_1, y_2 \cdots y_6$ are the value of y in the table.

Here, $a=0$, $b=6$, $h=1$

$\therefore$ Area of cross-section is $= \int_0^6 y\,dx$

By weddle's ~~Rul~~ rule, we have

$$\int_0^6 y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6\right]$$

$$= \frac{3}{10}\left[1 + (5\times 0.5) + 0.2 + (6\times 0.1) + 0.05888 + (5\times 0.0385) + (2\times 0.027)\right]$$

$$= \frac{3}{10}\times 4.6053$$

$$= 1.38159$$ (Ans.)

Ans to the Q no - 4

Here, $a=0$, $b=1$, we shall divide the interval into six equal parts

$$\therefore h = \frac{1-0}{6} = \frac{1}{6}$$

Now we find the values of $y = \sqrt{1-x^3}$ for each point of subdivision in the following in the following table,

x	$y = \sqrt{1-x^3}$
$x_0 = 0$	$y_0 = 1$
$x_1 = x_0 + h = 1/6$	$y_1 = 0.9977$
$x_2 = x_0 + 2h = 1/3$	$y_2 = 0.9813$
$x_3 = x_0 + 3h = 1/2$	$y_3 = 0.9354$
$x_4 = x_0 + 4h = 2/3$	$y_4 = 0.8389$
$x_5 = x_0 + 5h = 5/6$	$y_5 = 0.6491$
$x_6 = x_0 + 6h = 1$	$y_6 = 0$

(i) By simpson's 3/8 rule, we have

$$\int_0^1 \sqrt{1-x^3}\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{48}\left[(1+0)+3(0.9977+0.9813+0.8389+0.6491)+2(0.9354)\right]$$

$$= 0.82949$$

10

② By Trapezoidal rule, we have

$$\int_0^1 \sqrt{1-x^3}\,dx = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{12}\left[(1+0)+2(0.9977+0.9813+0.9354+0.8389+0.6491)\right]$$

$$= 0.81707$$

(Ans)

11