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Answer to the question no-1

Let $f(x) = x^4 - x - 10$

Here, $f(1) = -10 < 0$ and
$f(2) = 4 > 0$

$$2 \uparrow \; 1.85558 \; \downarrow 1$$

So, $f(x)=0$ has at least one root lies between 1 and 2

Since
$f(x) = x^4 - x - 10$

$\therefore f'(x) = 4x^3 - 1$

By Newton-Raphson method, we have

$$X_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$= \frac{4x_n^4 - \cancel{x_n} - x_n^4 + \cancel{x_n} + 10}{4x_n^3 - 1}$$

$$\therefore X_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (1)$$

Taking $x_0 = 1.9$ the relation (1) gives

For $n=0$, $x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1}$

$$\Rightarrow x_1 = \frac{3 \times (1.9)^4 + 10}{4 \times (1.9)^3 - 1}$$

$$\therefore x_1 = 1.8572$$

For $n=1$, $x_2 = \dfrac{3x_1^4+10}{4x_1^3-1}$

$$\Rightarrow x_2 = \frac{3\times(1.8572)^4+10}{4\times(1.8572)^3-1}$$

$$\therefore x_2 = 1.8556$$

For $n=2$, $x_3 = \dfrac{3x_2^4+10}{4x_2^3-1}$

$$= \frac{3\times(1.8556)^4+10}{4\times(1.8556)^3-1}$$

$$\therefore x_3 = 1.8556$$

Due to repetition of x_2 and x_3, we stop our work. Hence, the required root is 1.856 correct to three decimal places.

Answer to the question no-2

Given the system of equation

$$\left.\begin{aligned} 4x-y-2z&=15\\ -x+2y+3z&=5\\ 5x-7y+9z&=8 \end{aligned}\right\} \quad \cdots \quad \cdots \quad \cdots (1)$$

The system (1) is equivalent to

$$AX = B \quad \cdots \quad \cdots \quad \cdots \quad \cdots (2)$$

where,

$$A = \begin{bmatrix} 4 & -1 & -2 \\ -1 & 2 & 3 \\ 5 & -7 & 9 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 15 \\ 5 \\ 8 \end{bmatrix}$$

Now our aim is to reduce the augmented matrix of (1) to upper triangular matrix.

Now, the augmented matrix $(A \setminus B)$ is

$$\left(\begin{array}{ccc|c} 4 & -1 & -2 & 15 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right) \cdot r_1' = \frac{1}{4} r_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 7/4 & 5/2 & 35/4 \\ 0 & -23/4 & 23/2 & -43/4 \end{array}\right) \begin{array}{l} [\because r_2' = r_2 + r_1] \\ [\because r_3' = r_3 - 5r_1] \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & -1 & 2 & -43/23 \end{array}\right) \begin{array}{l} [\because r_2' = \frac{4}{7} r_2] \\ [\because r_3' = \frac{4}{23} r_3] \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 24/7 & 504/161 \end{array}\right) [\because r_3' = r_3 + r_2]$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 1 & 21/23 \end{array}\right) [\because r_3' = \frac{7}{24} r_3]$$

The reduced system is

$$x - \frac{1}{4}y - \frac{1}{2}z = \frac{15}{4}$$

$$y + \frac{10}{7}z = 5$$

$$z = \frac{21}{23}$$

Now we back substitution, we get

$$z = \frac{21}{23}$$

$$y = 5 - \frac{10}{\cancel{7}} \times \frac{\cancel{21}^{3}}{23}$$

$$= 5 - \frac{30}{23}$$

$$\therefore y = \frac{85}{23}$$

$$x = \frac{1}{4} \times \frac{85}{23} + \frac{1}{2} \times \frac{21}{23}$$

$$= \frac{85}{92} + \frac{21}{46}$$

$$= \frac{127}{92}$$

$$\therefore x = \frac{127}{92}, \quad y = \frac{85}{23}, \quad z = \frac{21}{23}$$

Answer to theq question no—3

Here

$x_0 = 0$, $x_1 = 1$, $x_2 = 3$, $x_3 = 8$

$y_0 = 1$, $y_1 = 3$, $y_2 = 13$, $y_3 = 123$

$x - x_0 = x$	$x_0 - x_1 = -1$	$x_0 - x_2 = -3$	$x_0 - x_3 = -8$
$x_1 - x_0 = 1$	$x - x_1 = x - 1$	$x_1 - x_2 = -2$	$x_1 - x_3 = -7$
$x_2 - x_0 = 3$	$x_2 - x_1 = 2$	$x - x_2 = x - 3$	$x_2 - x_3 = -8$
$x_3 - x_0 = 8$	$x_3 - x_1 = 7$	$x_3 - x_2 = 5$	$x - x_3 = x - 8$

By Lagranges interpolation formula, we have

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$\therefore y(5) = \frac{(5-1)(5-3)(5-8)}{(-1)\times(-3)\times(-8)} \times 1 + \frac{5\times(5-3)(5-8)}{1\times(-2)\times(-7)} \times 3 +$$

$$\frac{5(5-1)(5-8)}{3\times 2\times(-5)} \times 13 + \frac{5(5-1)(5-3)}{8\times 7\times 5} \times 123$$

$$= 1 - 6.4286 + 26 + 17.57143$$

$$= 38.14286$$

Answer to the question no - 4

Here, $a=0$, $b=1$ we shall divide the interval into six equal parts

$$\therefore h = \frac{1-0}{6} = \frac{1}{6}$$

Now we find the values of $y=\sqrt{1-x^3}$ for each point of subdivision in the following table

x	$y=\sqrt{1-x^3}$
$x_0=0$	$y_0=1$
$x_1=x_0+h=1/6$	$y_1=0.9977$
$x_2=x_0+2h=1/3$	$y_2=0.9813$
$x_3=x_0+3h=1/2$	$y_3=0.9354$
$x_4=x_0+4h=2/3$	$y_4=0.8389$
$x_5=x_0+5h=5/6$	$y_5=0.6491$
$x_6=x_0+6h=1$	$y_6=0$

① By simpson's 3/8 rule, we have

$$\int_0^1 \sqrt{1-x^3}\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{48}\left[(1+0)+3(0.9977+0.9813+0.8389+0.6491)+2(0.93\right.$$

$$= 0.82949$$

② By Trapezoidal rule, we have

$$\int_0^1 \sqrt{1-x^3}\,dx = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{12}\left[(1+0)+2(0.9977+0.9813+0.9354+0.8389+0.6491)\right]$$

$$= 0.81707$$

Answer to the question no-7

Represent the distance x along x-axis and y along y-axis then $y_0, y_1, y_2 \cdots y_6$ are the value of y in the table.

Here, $a=0$, $b=6$, $h=1$

$\therefore$ Area of cross-section is $= \int_0^6 y\,dx$

By weddle's Rule, we have

$$\int_0^6 y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6\right]$$

$$= \frac{3}{10}\left[1 + (5 \times 0.5) + 0.2 + (6 \times 0.1) + 0.0588 + (5 \times 0.0385) + (2 \times 0.027)\right]$$

$$= \frac{3}{10} \times 4.6053$$

$$= 1.38159$$