Victoria University
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# Final Exam Assessment


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## Course Title: Numerical Methods

**Course Code:** MAT 415

## Submitted To:

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## Answer to the question no: 1

**# Find a real root of $x^4 - x - 10 = 0$ by Newton—Raphson method.**

Let, $x^4 - x - 10$
So that,
$f(1) = -10 = (-)ve$
$f(2) = 16 - 2 - 10 = 4 = (+)ve$
So, the root of $f(x) = 0 = (-)ve$ lies $bet^n$ 1 & 2
Let's take $x_0 = 2$
Also $f'(x) = 4x^3 - 1$

Newton-Raphson formula is,
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Putting n=0, the 1st Approximation is,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 2 - \frac{f(2)}{f'(2)}$$
$$= 2 - \frac{4}{4 \text{ x } 2^3 - 1} = 1.871$$

Putting n=1, the 2nd Approximation is,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 1.871 - \frac{f(1.871)}{f'(1.871)} = 1.856$$

Putting n=2, the 3rd Approximation is,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$$

Here, $x_2 = x_3$
**So, the root is 0.63**

## Answer to the question no: 2

**# Solve the following by Gauss—Elimination method.**

5x - 7y + 9z = 8 …… (i)

4x - y + 2z = 15 …… (ii)

-x + 2y + 3z = 5 …… (iii)

For eliminating 'x' from (ii) and (iii) using (i),
We have to multiply (i) by 4 and (ii) by 5

(20x - 28y + 36z) - (20x - 5y + 10z) = 32 – 75

20x - 28y + 36z - 20x + 5y - 10z) = - 43

- 23y + 26z = - 43 …… (iv)

Now, we have to multiply (i) by 1 and (iii) by -5. Then the equations will be-

(5x - 7y + 9z) - (5x - 10y - 15z) = 8 – (-25)

5x - 7y + 9z - 5x + 10y + 15z = 8 +25

3y + 24z = 33 …… (v)

Now, the (iv) and (v) equations are,

- 23y + 26z = - 43 …… (iv)

3y + 24z = 33 …… (v)

For eliminating 'y' from equation (iv) and (v),
We have to multiply (iv) by -3 and (v) by 23

(69y - 78z) – (69y + 552z) = 129 – 759

- 78z - 552z = 129 – 759

-630z = - 630

z = 1

Now, put the value of z in (v)

3y + 24 x 1 = 33

3y = 33 – 24

$y = \frac{9}{3}$

y = 3

Now, put the value of y and z in (ii)

4x - y + 2z = 15

4x - 3 + 2 x 1 = 15

4x - 1= 15

4x = 15 + 1

x =16/4

x = 4

**So, x = 4, y = 3 & z = 1**

## Answer to the question no: 3

**# Use Lagrange's interpolation formula to find y when x = 5 from the following data**

| X | 0 | 1 | 3 | 8 |
|---|---|---|---|---|
| Y | 1 | 3 | 13 | 123 |

We know that,

$$y_x = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \text{ x } y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \text{ x } y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \text{ x } y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \text{ x } y_3$$

$$y_5 = \frac{(5-1)(5-3)\,(5-8)}{(0-1)(0-3)(0-8)} \text{ x } 1 + \frac{(5-0)(5-3)(5-8)}{(1-0)(1-3)(1-8)} \text{ x } 3 + \frac{(5-0)(5-1)(5-8)}{(3-0)(3-0)(3-8)} \text{ x } 13 + \frac{(5-0)(5-1)(5-3)}{(8-0)(8-1)(8-3)} \text{ x } 123$$

$$y_5 = \frac{(4)(2)\,(-3)}{(-1)(-3)(-8)} \text{ x } 1 + \frac{(5)(2)(-3)}{(1)(-2)(-7)} \text{ x } 3 + \frac{(5)(4)(-3)}{(3)(3)(-5)} \text{ x } 13 + \frac{(5)(4)(2)}{(8)(7)(5)} \text{ x } 123$$

$$y_5 = \frac{-24}{-24} \text{ x } 1 + \frac{-30}{14} \text{ x } 3 + \frac{-60}{-45} \text{ x } 13 + \frac{40}{280} \text{ x } 123$$

$$y_5 = 1 - 6.429 + 17.333 + 17.571$$

$$\boldsymbol{y_5 = 29.475\ (Ans.)}$$

## Answer to the question no: 4

(i) Simpson's rule-

$$\int_0^1 \sqrt{1-x^3}$$

| x | 0 | 0.16 | 0.32 | 0.48 | 0.64 | 0.8 | 0.96 |
|---|---|---|---|---|---|---|---|
| y | 1 | 0.998 | 0.984 | 0.943 | 0.859 | 0.699 | 0.12 |

We know that,

$$I = \frac{3h}{8}\ [X + 2T + 3R]$$

$X = y_0 + y_6 = 1 + 0.12 = 1.12$
$T = y_3 = 0.943$
$R = y_1 + y_2 + y_4 + y_5$
$R = 0.998 + 0.984 + 0.859 + 0.699 = 3.54$

$I = \frac{3}{8}$ x $0.16$ $[1.12 + (2 \text{ x } 0.943) + (3 \text{ x } 3.54)]$
$I = (0.375 \text{ x } 0.16)\ [1.12 + 1.886 + 10.62]$
$I = 0.06 \text{ x } 13.626$

$\boldsymbol{So, I = 0.82}$

**(ii) Trapezoidal rule-**

$$\int_{\mathbf{0}}^{\mathbf{1}} \sqrt{\mathbf{1 - x^3}}$$

n = 6
$\Delta x = \frac{1-0}{6} = 0.16$
We know that,
$x_i = a + i\Delta x$

So, $x_0 = 0$
$x_1 = 0.16$
$x_2 = 0.32$
$x_3 = 0.48$
$x_4 = 0.64$
$x_5 = 0.8$
$x_6 = 0.96$

Now,

$$\int_a^b f(x)dx = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$$\int_0^1 \sqrt{1 - x^3}\, dx = \frac{0.16}{2}[\sqrt{1 - 0^3} + 2\sqrt{1 - (0.16)^3} + 2\sqrt{1 - (0.32)^3}$$

$$+ 2\sqrt{1 - (0.48)^3} + 2\sqrt{1 - (0.64)^3 +} 2\sqrt{1 - (0.8)^3} + 2\sqrt{1 - (0.96)^3}$$

$$\int_0^1 \sqrt{1 - x^3}\, dx = 0.08\ [1\ + 1.98 + 1.96 + 1.78\ +\ 1.48\ +\ 0.98 + 0.24]$$

$$So, \int_0^1 \sqrt{1-x^3}\, dx = 0.7536$$

## Answer to the question no: 7

**# Find Solution usin Weddle's rule**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| y | 1 | .5 | .2 | .1 | .0588 | .0385 | .027 |

We know that,

$$I = \frac{3h}{10}\,[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

$$I = \frac{3 \text{ x } 1}{10}\,[1 + 5\,(0.5) + 0.2 + 6\,(0.1) + 0.0588 + 5\,(0.0385) + 0.027]$$

$$I = 0.3\,[1 + 2.5 + 0.2 + 0.6 + 0.0588 + 0.1925 + 0.027]$$

$$I = 0.3\,[4.5783]$$

***So*, *I*=1.37349**

**>>>>>>END<<<<<<**