Final Assessment
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CSE - 21st Batch
Computer Networks
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Ans. to the Q. no - 01

Let $f(x) = x^4 - x - 10$, ~~Here~~, Since $f(1) = -10 < 0$ and $f(2) = 4 > 0$

$$f(x) = x^4 - x - 10$$

$$\therefore f'(x) = 4x^3 - 1$$

By Newton–Raphson method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$= \frac{4x_n^4 - \cancel{x_n} - x_n^4 + \cancel{x_n} + 10}{4x_n^3 - 1}$$

$$\therefore x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1} \quad \cdots\cdots (1)$$

Taking $x_0 = 1.9$, the relation (1) gives

for $n = 0$, $x_1 = \dfrac{3x_0^4 + 10}{4x_0^3 - 1}$

$$\Rightarrow x_1 = \frac{3 \times (1.9)^4 + 10}{4 \times (1.9)^3 - 1}$$

$$\therefore x_1 = 1.8572$$

For $n=1$, $x_2 = \dfrac{3x_1^4+10}{4x_1^3-1}$

$\Rightarrow x_2 = \dfrac{3\times(1.8572)^4+10}{4\times(1.8572)^3-1}$

$\therefore x_2 = 1.85556$

For $n=2$, $x_3 = \dfrac{3x_2^4+10}{4x_2^3-1}$

$= \dfrac{3\times(1.85556)^4+10}{4\times(1.85556)^3-1}$

$\therefore x_3 = 1.85556$

Due to repetetion of x_2 and x_3, we stop our work hence the required root is 1.856 correct to three decimal.

Ans. to the Q. no - 2

given the system of equation

$$\left.\begin{aligned} 4x-y-2z&=15\\ -x+2y+3z&=5\\ 5x-7y+9z&=8 \end{aligned}\right\} \quad \text{-------- (1)}$$

The system (1) is equivalent to

$$AX = B \quad \text{..........} \quad (2)$$

Where,

$$A = \begin{bmatrix} 4 & -1 & -2 \\ -1 & 2 & 3 \\ 5 & -7 & 9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 15 \\ 5 \\ 8 \end{bmatrix}$$

now ~~of~~ our aim is to reduce the augmented metrix of (1) to upper triangular metrix.

now, the augmented metrix $(A \setminus B)$ is

$$\cancel{\left(\begin{array}{ccc|c} 4 & -1 & -2 & 15 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right)} \rightarrow \cancel{\left(\begin{array}{ccc} 1 & -\frac{1}{4} & -\frac{1}{2} \\ -1 & 2 & 3 \\ 5 & -7 & 9 \end{array}\right)}$$

$$\left(\begin{array}{ccc|c} 4 & -1 & -2 & 15 \\ 1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1/4 & 1/2 & 15/4 \\ -1 & 2 & 3 & 5 \\ 5 & -7 & 9 & 8 \end{array}\right) \quad r_1' = \frac{1}{4} r_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 7/4 & 5/2 & 35/4 \\ 0 & -23/4 & 23/2 & -43/4 \end{array}\right) \quad \begin{array}{l} [\because r_2' = r_2 + r_1] \\ [\because r_3' = r_3 - 5r_1] \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & -1 & 2 & -43/23 \end{array}\right) \quad \begin{array}{l} [\because r_2' = \frac{4}{7} r_2] \\ [\because r_3' = \frac{4}{23} r_3] \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 24/7 & 504/161 \end{array}\right) \quad [\because r_3' = r_3 + r_2]$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1/4 & -1/2 & 15/4 \\ 0 & 1 & 10/7 & 5 \\ 0 & 0 & 1 & 21/23 \end{array}\right) \quad [\because r_3' = \frac{7}{24} r_3]$$

The reduced system is

$$x = \frac{1}{4} y - \frac{1}{2} z = \frac{15}{4}$$

$$y + \frac{10}{7} z = 5, \quad \cancel{z = \frac{21}{23}}$$

$$z = \frac{21}{23}$$

now we back substitution, we get

$$z = \frac{21}{23}$$

$$y = 5 - \frac{10}{7} \times \frac{21}{23}$$

$$= 5 - \frac{30}{23}$$

$$\therefore y = \frac{85}{23}$$

$$x = \frac{1}{4} \times \frac{85}{23} + \frac{1}{2} \times \frac{21}{23}$$

$$= \frac{85}{92} + \frac{21}{46}$$

$$= \frac{127}{92}$$

$$\therefore x = \frac{127}{92},\ y = \frac{85}{23},\ z = \frac{21}{23}$$ Ans.

Bost

Ans. to the Q. no - 3

Here,

$x_0 = 0, \; x_1 = 1, \; x_2 = 3, \; x_3 = 8$

$y_0 = 1, \; y_1 = 3, \; y_2 = 13, \; y_3 = 123$

$x - x_0 = x$	$x_0 - x_1 = 1$	$x_0 - x_2 = -3$	$x_0 - x_3 = -8$
$x_1 - x_0 = 1$	$x - x_1 = x-1$	$x_1 - x_2 = -2$	$x_1 - x_3 = -7$
$x_2 - x_0 = 3$	$x_2 - x_1 = 2$	$x - x_2 = x-3$	$x_2 - x_3 = -8$
$x_3 - x_0 = 8$	$x_3 - x_1 = 7$	$x_3 - x_2 = 5$	$x - x_3 = x-8$

By langranges interpolation formula, we have -

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_2)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$\therefore y(5) = \frac{(5-1)(5-3)(5-8)}{(-1)\times(-3)\times(-8)} \times 1 + \frac{5\times(5-3)(5-8)}{1\times(-2)\times(-7)} \times 3 +$$

$$\frac{5(5-1)(5-8)}{3\times2\times(-5)} \times 13 + \frac{5(5-1)(5-3)}{8\times7\times5} \times 123$$

$$= 1 - 6.4286 + 26 + 17.57143$$

$$= 38.14286 \text{ Ans.}$$

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