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Ans to the Que No 1

$$\text{Let, } x = \frac{8}{9}$$

$$x' = 0.8889$$

Now,

(a) Absolute Error = $E_A = |x - x'|$

$$= \left| \frac{8}{9} - 0.8889 \right|$$
$$= \left| \frac{8 - 8.0001}{9} \right|$$
$$= 0.000011$$

(b) Relative Error = $E_R = \frac{E_A}{|x|}$

$$= \frac{\left| \frac{8}{9} - 0.8889 \right|}{\frac{8}{9}}$$
$$= \left| 1 - 1.0000125 \right|$$
$$= 0.0000125$$

(c) Percentage Error = $E_R \times 100$

$$= 0.0000125 \times 100$$
$$= 0.00125$$

Ans to the Que NO 3

let,

$$f(x) = x^4 - x - 10$$

$$\therefore f(-2) = (-2)^4 - (-2) - 10 = 8 > 0$$

$$f(-1) = (-1)^4 - (-1) - 10 = -8 < 0$$

$$\begin{array}{c} -2 \\ \uparrow \\ -1.6975 \\ \downarrow \\ -1 \end{array}$$

$\therefore f(-2)$ and $f(-1)$ are of opposite signs, so at least one root of the given equation lies between -2 and -1

Iteration - 1

$$\text{let, } x_0 = \frac{-2 - 1}{2} = -1.5$$

$$\therefore f(x_0) = f(-1.5) = (-1.5)^4 - (-1.5) - 10 = 3.4375 < 0$$

\therefore The root lies between -2 and -1.5

Iteration - 2

$$\text{let, } x_1 = \frac{-2 - 1.5}{2} = -1.75$$

$$\therefore f(x_1) = f(-1.75) = (-1.75)^4 - (-1.75) - 10 = 1.289 > 0$$

The root lies between -1.75 and -1.5

Iteration-3

$$\text{let } x_2 = \frac{-1.75 - 1.5}{2} = -1.625$$

$$\therefore f(x_2) = f(-1.625) = (-1.625)^4 - (-1.625) - 10 = -1.4021 < 0$$

The root lies between -1.75 and -1.625

Iteration-4

$$\text{let, } x_3 = \frac{-1.75 - 1.625}{2} = -1.6875$$

~~$$\therefore f(x_2) = f(-1.625)$$~~

$$\therefore f(x_3) = f(-1.6875) = (-1.6875)^4 - (-1.6875) - 10 = -0.2034 < 0$$

The root lies between -1.75 and -1.6875

Iteration 5

$$\text{let, } x_4 = \frac{-1.75 - 1.6875}{2} = -1.71875$$

$$\therefore f(x_4) = f(-1.71875) = (-1.71875)^4 - (-1.71875) - 10 = 0.4455 > 0$$

The root lies between -1.71875 and -1.6875

Iteration 6,

$$\text{let } x_5 = \frac{-1.71875 - 1.6875}{2} = -1.70313$$

$$\therefore f(x_5) = f(-1.70313) = (-1.70313)^4 - (-1.70313) - 10 = 0.1169 > 0$$

The root lies between -1.70313 and -1.6875

Iteration - 7,

$$\text{let } x_6 = \frac{-1.70313 - 1.6875}{2} = -1.69531$$

$$\therefore f(x_6) = f(-1.69531) = (-1.69531)^4 - (-1.69531) - 10 = -0.04438 < 0$$

The root lies ~~bet~~ between -1.70313 and -1.69531

Iteration - 8,

$$\text{let } x_7 = \frac{-1.70313 - 1.69531}{2} = -1.69922$$

$$\therefore f(x_7) = f(-1.69922) = (-1.69922)^4 - (-1.69922) - 10 = 0.03600 < 0$$

\therefore The required root lies between -1.69531 and -1.69922

From here it is evident that up to two decimal places the required root of the given equation is -1.69 .

P.T.O

Ans to the Que NO-04

Let $f(x) = x^3 - 4x - 9$

Since,

$$f(2) = (2)^3 - 4 \times 2 - 9 = -9$$

$$f(3) = (3)^3 - 4 \times 3 - 9 = 6$$

Here, $a = 2$, $b = 3$

$$\therefore f(a) \times f(b) = -9 \times 6 = -54 < 0$$

At least one root of the equation, lies between $[2, 3]$

a	b	f(a)	f(b)	$c = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)}$	f(c)
2	3	-9	6	2.6	-1.824
2.6	3	-1.824	6	2.693	-0.242
2.693	3	-0.242	6	2.704	-0.045
2.704	3	-0.045	6	2.706	-0.048

From here it is evident that upto two decimal places the required root of given equation is 2.70

Ans to the Que No - 05

$$\text{let, } f(x) = x^3 - 4x - 9 = 0$$

$$\Rightarrow x^3 - 4x - 9 = 0$$

$$\Rightarrow x^3 = 9 + 4x$$

$$\Rightarrow x = \sqrt[3]{4x + 9}$$

$$\Rightarrow x = g(x)$$

$$\text{where, } g(x) = \sqrt[3]{4x + 9}$$

$$\text{let } x_0 = 1$$

$$x_1 = g(x_0) = g(1) = (4 \times 1 + 9)^{1/3} = 2.3513$$

$$x_2 = g(x_1) = g(2.3513) = (4 \times 2.3513 + 9)^{1/3} = 2.6403$$

$$x_3 = g(x_2) = g(2.6403) = (4 \times 2.6403 + 9)^{1/3} = 2.6944$$

$$x_4 = g(x_3) = g(2.6944) = (4 \times 2.6944 + 9)^{1/3} = 2.7043$$

$$x_5 = g(x_4) = g(2.7043) = (4 \times 2.7043 + 9)^{1/3} = 2.7061$$

$$x_6 = g(x_5) = g(2.7061) = (4 \times 2.7061 + 9)^{1/3} = 2.7064$$

Due to repetition of x_5 and x_6 , we stop our work. Thus the required root is 2.706 correct to three decimal places.

Again,

$$\text{let } x_0 = 2$$

~~x_0~~

$$x_1 = g(x_0) = g(2) = (4 \times 2 + 9)^{1/3} = 2.571$$

$$x_2 = g(x_1) = g(2.571) = (4 \times 2.571 + 9)^{1/3} = 2.682$$

$$x_3 = g(x_2) = g(2.682) = (4 \times 2.682 + 9)^{1/3} = 2.702$$

$$x_4 = g(x_3) = g(2.702) = (4 \times 2.702 + 9)^{1/3} = 2.706$$

$$x_5 = g(x_4) = g(2.706) = (4 \times 2.706 + 9)^{1/3} = 2.706$$

Due to repetition of x_4 and x_5 we stop our work. Thus the required root is 2.706 correct to three decimal place.

Ans to the Que NO - 6 (i)

Converting the binary to decimal

$$(0111.0111)_2 = 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$

$$= 0 + 4 + 2 + 1 + 0 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

$$= 4 + 2 + 1 + 0.25 + 0.125 + 0.0625$$

$$= (7.4375)_{10}$$

Here is the answer, the binary number $(0111.0111)_2$ converted to decimal is therefore equal to $(7.4375)_{10}$

Ans to the Que NO - 6 (11)

Converting the hexadecimal number to decimal number

$$\begin{aligned}
 (F2C.A)_{16} &= F \times 16^2 + 2 \times 16^1 + C \times 16^0 + A \times 16^{-1} \\
 &= 15 \times 256 + 2 \times 16 + 12 \times 1 + 10 \times \frac{1}{16} \\
 &= 3840 + 32 + 12 + 0.625 \\
 &= (3884.625)_{10}
 \end{aligned}$$

Here is the final answer,

The Hexadecimal Number $(F2C.A)_{16}$ converted to decimal is therefore equal to $(3884.625)_{10}$.

END.