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Course Title : Differential Calculas and Coordinate Geometry

Course Code : MAT-115

①

Answer to the Question no - ①

* $$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

By using Rationalization

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$\lim_{x \to 0} \frac{(1-x) - 1}{x(\sqrt{1+x} + 1)}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{1+x} + 1)}$$

$$= \frac{1}{\sqrt{1} + 1}$$

$$= \frac{1}{2} \quad \text{(Ans.)}$$

(2)

Answer to the Question No - 2.

* $f(x) = x^3 + 5x^2$

We have to find 1) $F(1)$, 2) $F(-2)$, 3) $F(1/2)$

1) $F(1)$

$F(x) = x^3 + 5x^2$

Putting value of $x = 1$ in the function

$F(1) = (1)^3 + 5(1)^2$

$= 1 + 5$

$= 6$

2) $F(-2)$

$F(x) = x^3 + 5x^2$

Putting value of $x = -2$ in the function

$F(-2) = (-2)^3 + 5(-2)^2$

$= -8 + 20$

$= 12$

③

③ $F(1/2)$

$F(x) = x^3 + 5x^2$

Putting value of $x = 1/2$ in the function

$F(1/2) = (1/2)^3 + 5(1/2)^2$

$= 11/8$

∴ The value of $F(1)$, $F(-2)$ and $F(1/2)$ is 6, 12, 11/8. (Ans).

Answer to the Question No-3

* $\int (2e^x + \frac{6}{x} + \text{In } 2)\, dx$

$= 2\int e^x\, dx + 6 \int \frac{1}{x}\, dx + \text{In } 2 \int dx$

$= 2e^x + 6\ln|x| + (\text{In } 2)x + c.$ (Ans.)

④

Answer to the Question No - 4

$f(x) x^2 \sin x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 + h^2 + 2xh) \sin(x+h) - x^2 \sin x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 \sin(x+h) - x^2 \sin x}{h} + \lim_{h \to 0} \frac{h(h+2x) \sin(x+h)}{h}$$

$$= x^2 \lim_{h \to 0} \frac{2 \sin\left(\frac{x+h-x}{2}\right) \times \cos\left(\frac{x+h+x}{2}\right)}{h} + 2x \sin x$$

$$= x^2 \lim_{h \to 0} \frac{\sin(h/2)}{h/2} \times \cos\left(\frac{2x+h}{2}\right) + 2x \sin x$$

$$= x^2 . 1 . \cos x + 2x \sin x$$

$$= x^2 \cos x + 2x \sin x \text{ (Ans.)}$$

(5)

Answer to the Question No - 5.

In calculus, the chain rule is a formula that express the derivative of the composition of two differentiable functions f and g in terms of the derivatives of f and g. More precisely, if (h) = $f \circ g$ is the function such that $h(x) = f(g(x))$ for every x, then the chain rule is in Lagrange's notation,

$$h'(x) = f'(g(x))g'(x).$$

or, equivalently,

$$h' = (f \circ g)' = (f' \circ g) \cdot g'.$$

The chain rule may also be expressed in Leibniz's notation. If a variable z depends on the variable y, which itself depends on ~~x~~ as well, via the intermediate variable y. In

(6)

this case, the chain rule is expressed as

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx},$$

and

$$\left.\frac{dz}{dx}\right|_{x} = \left.\frac{dz}{dy}\right|_{y(x)} \cdot \left.\frac{dy}{dx}\right|_{x},$$

for indicating at which points the derivatives have to be evaluated.

In integration, the counterpart to the chain rule is the substitution rule.