Victoria University of Bangladesh

Course Title	:	*Differential Equation and Fourier Analysis*
Course Code	:	*MAT-325*
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Submit Date	:	*08/10/2022*
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Id Number	:	***2519150021***
Program	:	*CSIT (Day)*

Answer to the question number : 01

Initial Value Problems (IVP) : An initial value probem (IVP) is a problem where we want to find a slution to some differential equation that satisfies a given Initial value $y(x_0) = y_0$.

When we solve diffenential equation, often times we will obtain many if not infinitely many solutions. For example, consider the diffenensial equation $\frac{dy}{dx} = y$. All solutions to this differential equation are given as $y = Ce^x$, where c is a constant, we can venify this because $\frac{d}{dx}(Ce^x) = Ce^x$. However, suppose that instead we wanted to find a specific solution to our diffenential equation.

For example, suppose that, we look at $\frac{dy}{dx}=y$ and suppose that we also want such that $y(0) = Ce^0 = C$ and $C=3$. Therefore the solution $y = 3e^x$ both satisfies $\frac{dy}{dx}=y$ and $y(0)=3$. This is what we essentially call an initial value problem where $y(0)=3$ is the initial value.

Answer to the question number : 02

Here,

$$x^2y^2 + xy = x^2+3; \quad y(0) = -2$$

From here we can write, $x=0$ and $y=-2$

We know that, $y' + Py = Q$

We have

$$\frac{x^2y'}{x^2} + \frac{xy}{x^2} = \frac{x^2}{x^2} + \frac{3}{x^2} \text{ (by dividing } x^2\text{)}$$

$$\Rightarrow y' + \frac{1}{x}y = 1 + \frac{3}{x^2}$$

Now,

$$\mu(x) = e^{\int P(x)dx} = e^{\int \left(\frac{1}{x}\right)dx} = e^{\log x}$$
$$= x$$

Multiplying the given equation with $\mu(x)$

We have,

$$\cancel{\frac{x^2y'}{x^2} + \frac{xy}{x^2} = \frac{x^2}{x^2} + \frac{3}{x^2}}$$ ~~(by diving~~

$$x\left(y' + \frac{1}{x}y\right) = x\left(1 + \frac{3}{x^2}\right)$$
$$= xy' + y = x + \frac{3}{x}$$
$$= \frac{d(yx)}{dx} = x + \frac{3}{x}$$

Integrating both sides with respect to x,

we get, $\int\left[\frac{d(yx)}{dx}\right] \cdot dx = \int \left(x + \frac{3}{x}\right) dx$

$$\Rightarrow yx = \frac{x^2}{2} - \frac{3}{2x^2} + c$$
$$\Rightarrow y = \frac{\frac{x^2}{2} - \frac{3}{2x^2} + c}{x}$$

$$\Rightarrow y = \frac{x^2}{2} - \frac{3}{2x^2} + c \cdot x^{-1} \longrightarrow (1)$$

$$\Rightarrow -2 = \frac{0}{2} - \frac{3}{2(0)^2} + c(0)^{-1}$$

$$\Rightarrow -2 = 3 + c$$

$$\Rightarrow c = -5$$

Now,

Put the value of c In the equ^n (1)

$$\Rightarrow y = \frac{x^2}{2} - \frac{3}{2x^2} + (-5) . x^{-1}$$

$$\Rightarrow \frac{x^2}{2} - \frac{3}{2x^2} - 5x^{-1}$$

Ans

Answer to the question number : 03

Boundary value Problems (BVP) are similar to initial value Problem (BVP). A boundary value problems has conditions specified at the extrems (boundaries) of the independent variable in the equation whereas an initial value problem has all of the conditions specified at the same value of the independent variable that value is at the lower boundary of the domain, thus the term 'initial' value). A boundary value is a data value that corresponds to a minimum or maximum input, internal, or output value specified for a system or component.

For example, if the independent variable is time over the domain $[0, 1]$, a boundary value problem would specify values for $y(t)$ at both $t=0$ and $t=1$; whereas an initial value problem specify a value of $y(t)$ and $y'(t)$ at time $t=0$

Answer to the question number : 04

$$y'' - 18y' + 77y = 0; \quad y(0) = 4, \; y'(0) = 8$$

From here we can write,

$$x = 0 \text{ and } y = 4 \; (\text{for } y(0) = 4)$$

$$\text{and } x = 0 \text{ and } y' = 8$$

We have,

$$m^2 - 18m + 77 = 0 \quad (\text{by using axiliary equation})$$

$$= m^2 - 11m - 7m + 77 = 0$$

$$= m(m-11) - 7(m-11) = 0$$

$$= (m-11)(m-7) = 0$$

So we can write,

$$m - 11 = 0 \Rightarrow m = 11 \qquad \Big| \qquad m - 7 = 0 \Rightarrow m = 7$$

We know,

$$y = c_1 e^{mx} + c_2 e^{mx}$$

$$\Rightarrow y = c_1 e^{11x} + c_2 e^{7x} \longrightarrow (1)$$

$\cancel{\Rightarrow y =}$

Now, Put the value of $x=0$ and $y=4$ in the equ^n — (1)

$$\Rightarrow 4 = c_1 e^{11.0} + c_2 e^{7.0}$$

$$\Rightarrow 4 = c_1 e^0 + c_2 e^0$$

$$\Rightarrow c_1 + c_2 = 4 \longrightarrow (ii)$$

Now, $y' = 11 c_1 e^{11x} + 7c_2 e^{7x}$ [derivative of equ^n (1)] --- (iii)

Now put the value of $x=0$ and $y'=8$ in the equ^n (iii)

$$\Rightarrow 8 = 11 c_1 e^{11.0} + 7c_2 e^{7.0}$$

$$\Rightarrow 8 = 11 c_1 e^0 + 7c_2 e^0$$

$$\Rightarrow 11 c_1 + 7c_2 = 8 \longrightarrow (iv)$$

$\Rightarrow 11c_1 = 8 - 7c_2$

$\Rightarrow c_1 = \frac{8 - 7c_2}{11} \longrightarrow$ ⑧

Now put the value of c_1, in the equ $\rightarrow$ ①

$\Rightarrow \frac{8-7c_2}{11} + c_2 = 4$

$\Rightarrow \frac{8 - 7c_2 + 11c_2}{11} = 4$

$\Rightarrow \frac{8 + 4c_2}{11} = 4$

$\Rightarrow 8 + 4c_2 = 44$

$\Rightarrow 4c_2 = 44 - 8$

$\Rightarrow c_2 = \frac{36}{4}$

$\therefore c_2 = 9$

Now put the ~~equation~~ value of c_2 in the equ ⑤

$c_1 = \frac{8 - 7(9)}{11}$

$\Rightarrow c_1 = \frac{8 - 63}{11}$

$\Rightarrow c_1 = \frac{-55}{11}$

$\therefore c_1 = -5$

Now ~~the~~

Put the value of c_1 and c_2 in the eque-(1)

$y = -5e^{11x} + 9e^{7x}$ Ans

$y' = 11(-5)e^{11x} + 7(9)e^{7x}$

$\therefore y' = -55e^{11x} + 63e^{7x}$ Ans

Answer to the question num : 5

$6y'' - 5y' + y = 0$; $y(0)=4$, $y'(0)=0$

From here, we can write,

$x=0$ and $y=4$ (for $y(0)=4$)

and $x=0$ and $y'=0$ (for $y'(0)=0$)

Now,

$$6m^2 - 5m + 1 = 0$$

$$\Rightarrow 6m^2 - 3m - 2m + 1 = 0$$

$$\Rightarrow 3m(2m-1) - (2m-1) = 0$$

$$\Rightarrow (2m-1)(3m-1) = 0$$

We can write,

$$2m - 1 = 0 \qquad \Big| \qquad 3m - 1 = 0$$

$$\therefore m = \frac{1}{2} \qquad \Big| \qquad m = \frac{1}{3}$$

we know,

$$y = c_1 e^{mx} + c_2 e^{mx}$$ [for, Roots are real and differ]

$$\Rightarrow y = c_1 e^{\frac{1}{2}x} + c_2 e^{\frac{1}{3}x} \longrightarrow (1)$$

Now put the value of $x=0$ and $y=4$ in the equ^n $\longrightarrow$ (i)

$$\Rightarrow 4 = c_1 e^{\frac{1}{2}\cdot 0} + c_2 e^{\frac{1}{3}\cdot 0}$$

$$\Rightarrow 4 = c_1 e^0 + c_2 e^0$$

$$\Rightarrow c_1 + c_2 = 4 \longrightarrow \text{(ii)}$$

Now, $y' = \frac{1}{2} c_1 e^{\frac{1}{2}x} + \frac{1}{3} c_2 e^{\frac{1}{3}x} \longrightarrow$ (iii)

[derivative of equ^n - (i)]

~~Now put the value of x=0 in the e.~~

Now put the value of $x=0$ and $y'=0$ in the equ^n $\longrightarrow$ (iv)

$$\Rightarrow 0 = \frac{1}{2} c_1 e^{\frac{1}{2}\cdot 0} + \frac{1}{3} c_2 e^{\frac{1}{3}\cdot 0}$$

$\Rightarrow 0 = -\frac{1}{2} c_1 e^0 + \frac{1}{3} c_2 e^0$

$\Rightarrow 0 = \frac{1}{2} c_1 + \frac{1}{3} c_2 e^0$

$\Rightarrow -\frac{1}{2} c_1 + \frac{1}{3} c_2 = 0$

$\Rightarrow \frac{1}{2} c_1 = -\frac{1}{3} c_2$

$\Rightarrow c_1 = -\frac{2}{3} c_2 \rightarrow (N)$

Now put the value of c_1 in the equ $\rightarrow$ (ii)

$-\frac{2}{3} c_2 + c_2 = 4$

$\Rightarrow \frac{c_2}{3} = 4$

$= c_2 = 12$

Now put the value of c_2 in the equ – (N)

$c_1 = -\frac{2}{3} \times 12$

$\Rightarrow c_1 = -\frac{24}{3}$

$\therefore c_1 = -8$

Now,

Put the value of c_1 and c_2 in the equ - ①×①

$\# \; y = -8e^{\frac{1}{2}x} + 12e^{\frac{1}{3}x}$ Ans:

$\# \; y' = -\frac{8}{2} e^{\frac{1}{2}x} + \frac{12}{3} e^{\frac{1}{3}x}$

$\Rightarrow y' = -4e^{\frac{1}{2}x} + 4e^{\frac{1}{3}x}$

Ans