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Differential Equation and Fourier Analysis

MAT-325

Final

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Answer to the Question No- 1

Initial Value Problems (IVP): An Initial Value Problems (IVP) is a Problem where we want to find a solution to some differential equation that satisfies a given Initial value $y(x_0) = y_0$.

When we solve differential equations, often times we will obtain many if not infinitely many solutions. For example, consider the differential equation $\frac{dy}{dx} = y$. All solutions to this differential equation are given as $y = Ce^{x}$, where C is a constant, We can verify this because $\frac{d}{dx}(Ce^{x}) = Ce^{x}$. However, suppose that instead we wanted to find a specific solution to our differential equation.

For example, suppose that we look at $\frac{dy}{dx} = y$ again, and suppose that we also want y such that $y(0) = 3$. Since our solution set is $y = Ce^{x}$, we see that $3 = y(0) = Ce^{0} = C$ and so $C = 3$. Therefore the solution $y = 3e^{x}$ both satisfies $\frac{dy}{dx} = y$ and $y(0) = 3$. This is what we essentially call an initial value problem where $y(0) = 3$ is the initial value.

Answer to the Question No- 2

$x^2y' + xy = x^2 + 3$; $y(0) = -2$

From here we can write, $x=0$ and $y=-2$

We know, $y' + Py = Q$

We have,

$$\frac{x^2y'}{x^2} + \frac{xy}{x^2} = \frac{x^2}{x^2} + \frac{3}{x^2} \quad \text{(by dividing } x^2\text{)}$$

$$\Rightarrow y' + \frac{1}{x}y = 1 + \frac{3}{x^2}$$

Now, $\mu(x) = e^{\int P(x)\,dx} = e^{\int(\frac{1}{x})\,dx} = e^{\ln x} = x$

Multiplying the given equation with $\mu(x)$,

we have, $x(y' + \frac{1}{x}y) = x(1 + \frac{3}{x^2})$

$$\Rightarrow xy' + y = x + \frac{3}{x}$$

$$\Rightarrow \frac{d(yx)}{dx} = x + \frac{3}{x}$$

Integrating both sides with respect to x,

we get, $\int\left[\frac{d(yx)}{dx}\right]dx = \int\left(x + \frac{3}{x}\right)dx$

$$\Rightarrow yx = \frac{x^2}{2} - \frac{3}{2x^2} + c$$

$$\Rightarrow y = \frac{\frac{x^2}{2} - \frac{3}{2x^2} + c}{x}$$

$$\Rightarrow y = \frac{x^2}{2} - \frac{3}{2x^2} + c.x^{-1} \quad \cdots\cdots (1)$$

$$\Rightarrow -2 = \frac{0}{2} - \frac{3}{2(0)^2} + c(0)^{-1}$$

$$\Rightarrow -2 = 3 + c$$

$$\Rightarrow c = -5$$

Now, put the value of c in the eq^n (1)

$$\Rightarrow y = \frac{x^2}{2} - \frac{3}{2x^2} + (-5).x^{-1}$$

$$\Rightarrow \frac{x^2}{2} - \frac{3}{2x^2} - 5x^{-1}$$

Ans.

Answer to the Question No- 3

Boundary Value Problems (BVP) are similar to Initial Value Problems (IVP). A boundary value Problems has conditions specified at the extremes (boundaries) of the independent variable in the equation whereas an initial value problem has all of the conditions specified at the same value of the independent variable (and that value is at the lower boundary of the domain, thus the term "initial" value).
A boundary value is a data value that corresponds to a minimum or maximum input, internal, or output value specified for a system or component.

For example- if the independent variable is time over the domain [0,1], a boundary value problem would specify values for $y(t)$ at both $t=0$ and $t=1$, whereas an initial value problem would specify a value of $y(t)$ and $y'(t)$ at time $t=0$.

Answer to the Question No- 4

$y'' - 18y' + 77y = 0$; $y(0) = 4$, $y'(0) = 8$

From here we can write, $x = 0$ and $y = 4$ (for $y(0) = 4$)
and $x = 0$ and $y' = 8$

We have,
$m^2 - 18m + 77 = 0$ (by using auxilary equation)
$\Rightarrow m^2 - 11m - 7m + 77 = 0$
$\Rightarrow m(m-11) - 7(m-11) = 0$
$\Rightarrow (m-11)(m-7) = 0$

So, we can write,
$m - 11 = 0$
$\Rightarrow m = 11$

$m - 7 = 0$
$\Rightarrow m = 7$

we know,

$$y = c_1 e^{mx} + c_2 e^{mx} \quad [\text{For, Roots are real or different}]$$

$$\Rightarrow y = c_1 e^{11x} + c_2 e^{7x} \quad \text{---} \; (i)$$

Now, put the value of $x = 0$ and $y = 4$ in the equ^n (i)

$$\Rightarrow 4 = c_1 e^{11.0} + c_2 e^{7.0}$$

$$\Rightarrow 4 = c_1 e^0 + c_2 e^0$$

$$\Rightarrow c_1 + c_2 = 4 \quad \text{---} \; (ii)$$

Now, $y' = 11c_1 e^{11x} + 7c_2 e^{7x}$ [derivative of equ^n (i)]

--- (iii)

Now, put the value of $x = 0$ and $y' = 8$ in the equ^n (iii)

$$\Rightarrow 8 = 11c_1 e^{11.0} + 7c_2 e^{7.0}$$

$$\Rightarrow 8 = 11c_1 e^0 + 7c_2 e^0$$

$$\Rightarrow 11c_1 + 7c_2 = 8 \quad \text{---} \; (iv)$$

$$\Rightarrow 11c_1 = 8 - 7c_2$$

$$\Rightarrow c_1 = \frac{8 - 7c_2}{11} \quad \text{---} \; (v)$$

Now, put the value of c_1 in the equⁿ – (ii)

$$\Rightarrow \frac{8-7c_2}{11} + c_2 = 4$$

$$\Rightarrow \frac{8-7c_2+11c_2}{11} = 4$$

$$\Rightarrow \frac{8+4c_2}{11} = 4$$

$$\Rightarrow 8+4c_2 = 44$$

$$\cancel{\Rightarrow 4c_2} \Rightarrow 4c_2 = 44-8$$

$$\Rightarrow c_2 = \frac{36}{4}$$

$$\Rightarrow c_2 = 9$$

Now, put the value of c_2 in the equⁿ — (v)

$$\Rightarrow c_1 = \frac{8-7(9)}{11}$$

$$\Rightarrow c_1 = \frac{8-63}{11}$$

$$\Rightarrow c_1 = \frac{-55}{11}$$

$$\Rightarrow c_1 = -5$$

Now, put the value of c_1 and c_2 in the equⁿ — (i) & (ii)

$$\# y = -5e^{11x} + 9e^{7x} \quad \text{Ans}$$

~~Now, Put the vo~~

$$\# y' = 11(-5)\, e^{11x} + 7(9)\, e^{7x}$$

$$\Rightarrow y' = -55e^{11x} + 63e^{7x} \quad \text{Ans.}$$

Answer to the Question No- 5

$6y'' - 5y' + y = 0$; $y(0) = 4$, $y'(0) = 0$

From here we can write, $x=0$ and $y=4$ (for $y(0)=4$)
and, $x=0$ and $y'=0$ (for $y'(0)=0$)

Now,
$$6m^2 - 5m + 1 = 0$$
$$\Rightarrow 6m^2 - 3m - 2m + 1 = 0$$
$$\Rightarrow 3m(2m-1) - (2m-1) = 0$$
$$\Rightarrow (2m-1)(3m-1) = 0$$

So we can write

$$2m - 1 = 0$$
$$\Rightarrow m = \frac{1}{2}$$

$$3m - 1 = 0$$
$$\Rightarrow m = \frac{1}{3}$$

We know,
$$y = c_1 e^{mx} + c_2 e^{mx}$$ [For, Roots are real & different]
$$\Rightarrow y = c_1 e^{\frac{1}{2}x} + c_2 e^{\frac{1}{3}x} \text{ ——— (i)}$$

Now, Put the value of $x=0$ and $y=4$ in the eqn — (i)

$$\Rightarrow 4 = c_1 e^{\frac{1}{2}\cdot 0} + c_2 e^{\frac{1}{3}\cdot 0}$$
$$\Rightarrow 4 = c_1 e^0 + c_2 e^0$$
$$\Rightarrow c_1 + c_2 = 4 \text{ ——— (ii)}$$

Now, $y' = \frac{1}{2} c_1 e^{\frac{1}{2}x} + \frac{1}{3} c_2 e^{\frac{1}{3}x}$ [derivative of eqn — (i)] — (iii)

Now, put the value of $x=0$ and $y'=0$ in the equ^n — (III)

$$\Rightarrow 0 = \frac{1}{2}c_1 e^{1/2\cdot 0} + \frac{1}{3}c_2 e^{1/3\cdot 0}$$

$$\Rightarrow 0 = \frac{1}{2}c_1 e^0 + \frac{1}{3}c_2 e^0$$

$$\Rightarrow 0 = \frac{1}{2}c_1 + \frac{1}{3}c_2$$

$$\Rightarrow \frac{1}{2}c_1 + \frac{1}{3}c_2 = 0$$

$$\Rightarrow \frac{1}{2}c_1 = -\frac{1}{3}c_2$$

$$\Rightarrow c_1 = -\frac{2}{3}c_2 \quad \text{— (IV)}$$

Now, put the value of c_1 in the equ^n — (II)

$$-\frac{2}{3}c_2 + c_2 = 4$$

$$\Rightarrow \frac{c_2}{3} = 4$$

$$\Rightarrow c_2 = 12$$

Now, put the value of c_2 in the equ^n — (IV)

$$c_1 = -\frac{2}{3} \times 12$$

$$\Rightarrow c_1 = -\frac{24}{3}$$

$$\Rightarrow c_1 = -8$$

Now, put the value of c_1 and c_2 in the equ^n — (I) & (VII)

$$\# \; y = -8e^{1/2x} + 12e^{1/3x} \quad \underline{\text{Ans}}$$

$$\# \; y' = -\frac{8}{2}e^{1/2x} + \frac{12}{3}e^{1/3x}$$

$$\Rightarrow y' = -4e^{1/2x} + 4e^{1/3x} \quad \underline{\text{Ans}}$$

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