Victoria University
of Bangladesh

Assignment Topic:

Final Assessment

Course Title: Deferential Equation and Fourier Analysis

Course Code: MAT-325

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<u>Answer to the question no: 01</u>

<u>IVP</u>: IVP is short form of Initial Value Problem. In shortly we can say, a differential equation along with a sufficient number of initial conditions is called an Initial Value Problem (IVP). So, an Initial Value Problem (IVP) is a differential equation along with an appropriate number of initial conditions.

Calculus is a type of mathematics that focuses on rates of change, differential calculus, and the summation of many small pieces to make a whole, integral calculus. Initial value problems describe a type of problem in calculus. Initial value problems in calculus concern differential equations with a known initial condition that specifies the value of the function at some point. The purpose of these problems is to find the function that describes the system, which can be done by integrating the differential equation.

When performing an integration with an indefinite integral, or with no initial conditions, there is always an unknown constant. This is known as the constant of integration and is often denoted by C. An example is shown in the diagram.

$$\int 2x + 1\, dx = x^2 + x + C$$

In the absence of initial conditions, it is not possible to determine the value of this unknown constant. More information is needed to find the value of the constant of integration. Consider the same problem above if initial conditions are provided. Assume the initial conditions are

y(0)=2

This means that at x=0, the value of the function is 2, y=2. To determine the constant of integration, these values can be substituted into the function obtained from the integration.

Answer to the question no: 02

Product Rule: Product rule in calculus is a method used to find the derivative of any function given in the form of a product obtained by the multiplication of any two differentiable functions. The product rule in words states that the derivative of a product of two differentiable functions is equal to the sum of the product of the second function with differentiation of the first function and the product of the first function with the differentiation of the second function. That means if we are given a function of the form: f(x)·g(x), we can find the derivative of this function using the product rule derivative as,

$\frac{d}{dx} f(x)\cdot g(x) = [g(x) \times f'(x) + f(x) \times g'(x)]$

Product Rule Formula: We can calculate the derivative or evaluate the differentiation of the product of two functions using the product rule formula in Calculus. The product rule formula is given as,

$\frac{d}{dx} f(x) = \frac{d}{dx} \{u(x)\cdot v(x)\} = [v(x) \times u'(x) + u(x) \times v'(x)]$

where,

f(x) = Product of differentiable functions u(x) and v(x)

u(x), v(x) = Differentiable functions

u'(x) = Derivative of function u(x)

v'(x) = Derivative of the function v(x)

Example: Find the derivative of x· cos(x) using the product rule formula.

Solution:

Let f(x) = cos x and g(x) = x.

$\Rightarrow f'(x) = -\sin x$

$\Rightarrow g'(x) = 1$

$\Rightarrow [f(x)g(x)]' = [g(x)f'(x) + f(x)g'(x)]$

$\Rightarrow [f(x)g(x)]' = [(x\bullet(-\sin x) + \cos x\bullet(1)]$

$\Rightarrow [f(x)g(x)]' = - x \sin x + \cos x$

Answer: The derivative of x cos x using product rule is (- x sin x + cos x).

Math Solution: $x^2y' + xy = x^2 +3$; $y(0) = -2$

Solution:

$$x^2y' + xy = x^2 + 3 \quad ; \quad y(0) = -2$$

Here, $x = 0$ and $y = -2$

We know, $y' + P(x)y = Q(x)$

Now, by dividing by x^2 we find

$$\frac{x^2y'}{x^2} + \frac{xy}{x^2} = \frac{x^2}{x^2} + \frac{3}{x^2}$$

$$\Rightarrow y' + \frac{1}{x}y = 1 + \frac{3}{x^2}$$

Now, $\mu(x) = e^{\int P(x)dx} = e^{\int (\frac{1}{x})dx} = e^{\ln x} = x$

Now, multiplying the given equation with $\mu(x)$,

We have,

$$x\left(y' + \frac{1}{x}y\right) = x\left(1 + \frac{3}{x^2}\right)$$

$$\Rightarrow xy' + y = x + \frac{3}{x}$$

$$\Rightarrow \frac{d(xy)}{dx} = x + \frac{3}{x}$$

Integrating both side with respect to x, we get

$$\Rightarrow \int \left[\frac{d(xy)}{dx}\right] dx = \int \left(x + \frac{3}{x}\right) dx$$

$$\Rightarrow \int d(xy) = \int x\, dx + \int \frac{3}{x}\, dx$$

$$\Rightarrow xy = \frac{1}{2}x^2 + 3\ln x + c \quad \ldots\ldots\ldots (1)$$

$$\Rightarrow y = \frac{x}{2} + \frac{3\ln x + c}{x} \quad \ldots\ldots (11) \quad [\text{Dividing by } x]$$

From equation (1) we get

$$c = xy - \frac{1}{2}x^2 - 3\ln x$$

$$= 0 \cdot (-2) - \frac{1}{2} \cdot (0)^2 - 3\ln 0 \quad [\text{Replacing the value}]$$

Here, if we put value for $y(0) = -2$ then

$\ln x$ will be undefined.

So, from the equation (11) we get

$$y = \frac{x}{2} + \frac{3\ln x + c}{x}$$ where c - contant, y
$y = y(x)$.

<u>Ans:</u>

<u>Answer to the question no: 03</u>

Initial Value Problems:

Initial value problem does not require to specify the value at boundaries, instead it needs the value during initial condition. This usually apply for dynamic system that is changing over time as in Physics. An example, to solve a particle position under differential equation, we need the initial position and also initial velocity. Without these initial values, we cannot determine the final position from the equation. In initial value problems, we are given the value of function y(x) and its derivative y′(x) at the same point (initial point) sy at x=0 i.e y(0)=xi1 and $y'(0)=x_2$.

Boundary Value Problems:

In contrast, boundary value problems not necessarily used for dynamic system. Instead, it is very useful for a system that has space boundary. An example would be shape from shading problem in computer vision. To determine surface gradient from the PDE, one should impose boundary values on the region of interest. In boundary value problem, we are given the value of function y(x) at two different points, i.e y(a)=x1 and y(b)=x2.

So, we can say BVP is different from IVP.

Answer to the question no: 04

Solution:

Solution: Let $y'' - 18y' + 77y = 0$ .... (1)

and from $y(0) = 4$, we can write $x = 0, y = 4$

also from $y'(0) = 8$, we can write $x = 0, y' = 8$.

Now, (1) in auxilary equation form we can

write, $m^2 - 18m + 77 = 0$

$$\Rightarrow m^2 - 11m - 7m + 77 = 0$$

$$\Rightarrow m(m-11) - 7(m-11) = 0$$

$$\Rightarrow (m-11)(m-7) = 0$$

So, we can write,

$m - 11 = 0$ and $m - 7 = 0$

$\therefore m = 11$ $\therefore m = 7$

$\because$ Roots are real and different, so we can write,

$$y = c_1 e^{mx} + c_2 e^{mx}$$

$$\therefore y = c_1 e^{11x} + c_2 e^{7x} \text{ .... (ii)}$$

Now we will use initial conditions

put $x = 0$ and $y = 4$ in equation (ii)

$$4 = c_1 e^{11.0} + c_2 e^{7.0}$$

$$= c_1 e^0 + c_2 e^0$$

$$\therefore 4 = c_1 + c_2 \text{ .... (iii)}$$

Now, do the derivative of equation (ii)

$$y' = 11c_1e^{11x} + 7c_2e^{7x} \quad \cdots\cdots\cdots \text{(iv)}$$

Now, put the value of $y' = 8$ and $x = 0$ in equation (iv)

$$8 = 11c_1e^{11.0} + 7c_2e^{7.0}$$

$$\Rightarrow 8 = 11c_1e^{0} + 7c_2e^{0}$$

$$\Rightarrow 8 = 11c_1 + 7c_2 \quad \cdots\cdots\cdots \text{(v)}$$

Now, from equation (v) we get,

$$11c_1 = 8 - 7c_2$$

$$\therefore c_1 = \frac{8-7c_2}{11} \quad \cdots\cdots\cdots \text{(vi)}$$

Now, put the value of c_1 in equation (iii)

$$4 = \frac{8-7c_2}{11} + c_2$$

$$\Rightarrow 4 = \frac{8-7c_2+11c_2}{11}$$

$$\Rightarrow 4 = \frac{8+4c_2}{11}$$

$$\Rightarrow 8+4c_2 = 44$$

$$\Rightarrow 4c_2 = 44-8$$

$$\Rightarrow c_2 = 9$$

Put the value of c_2 in equation (vi),

$$\therefore\ c_1 = \frac{8-7c_2}{11}$$

$$= \frac{8-7\times 9}{11}$$

$$= \frac{8-63}{11}$$

$$= \frac{-55}{11}$$

$$= -5$$

So, $c_1 = -5$ and $c_2 = 9$

Now, put the value in equation (ii),

$$y = c_1 e^{11x} + c_2 e^{7x}$$

$$= -5e^{11x} + 9e^{7x}$$

Ans:

Put the value in equation (iv),

$$y' = 11c_1 e^{11x} + 7c_2 e^{7x}$$

$$= -55e^{11x} + 63e^{7x}$$

Ans:

Answer to the question no: 05

Solution:

Solution: Let $6y'' - 5y' + y = 0$ ....(i)

and from $y(0)=4$, we can write $x=0, y=4$

also from $y'(0)=0$, we can write $x=0, y'=0$

Now, (i) in auxilary equation form we can write, $6m^2 - 5m + 1 = 0$

$$\Rightarrow 6m^2 - 3m - 2m + 1 = 0$$

$$\Rightarrow 3m(2m-1) - 1(2m-1) = 0$$

$$\Rightarrow (2m-1)(3m-1) = 0$$

So, we can write,

$2m - 1 = 0$ and $3m - 1 = 0$

$\therefore m = \frac{1}{2}$ $\therefore m = \frac{1}{3}$

we can write general solution,

$$y = c_1 e^{\frac{1}{2}x} + c_2 e^{\frac{1}{3}x} \text{ .... (ii)}$$

Now, we will use initial conditions

Put $x=0$, $y=4$ in the equation (ii)

$$4 = c_1 e^{\frac{1}{2}\cdot 0} + c_2 e^{\frac{1}{3}\cdot 0}$$

$$\therefore 4 = c_1 + c_2 \text{ .... (iii)}$$

Now do the derivative of equation (ii)

$$y' = \frac{1}{2}c_1 e^{\frac{1}{2}x} + \frac{1}{3}c_2 e^{\frac{1}{3}x} \quad \ldots\ldots\ldots \text{(iv)}$$

Put the value of $y' = 0$ and $x = 0$ in equation (iv)

$$0 = \frac{1}{2}c_1 e^0 + \frac{1}{3}c_2 e^0$$

$$\Rightarrow 0 = \frac{1}{2}c_1 + \frac{1}{3}c_2$$

$$\Rightarrow \frac{1}{2}c_1 = -\frac{1}{3}c_2$$

$$\therefore c_1 = -\frac{2}{3}c_2 \quad \ldots\ldots\ldots \text{(v)}$$

Now put the value of c_1 in equation (iii)

$$4 = -\frac{2}{3}c_2 + c_2$$

$$\Rightarrow 4 = \frac{c_2}{3}$$

$$\Rightarrow c_2 = 12$$

Put the value in equation (v)

$$c_1 = -\frac{2}{3} \times 12$$

$$\therefore c_1 = -8$$

Now, put the values in equation (ii)

$$y = -8e^{\frac{1}{2}x} + 12e^{\frac{1}{3}x}$$

Now, put the values in equation (iv)

$$y' = -4e^{\frac{1}{2}x} + 4e^{\frac{1}{3}x}$$

Ans…